Chapter 34

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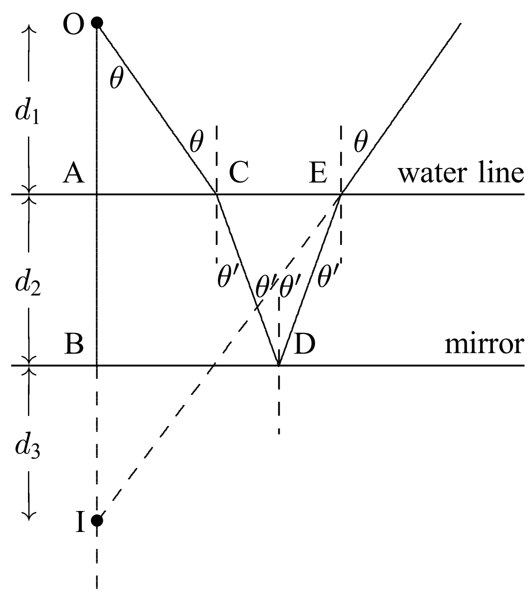

The light bulb is labeled O and its image is labeled I on the digram to the right. Consider the two rays shown on the diagram to the right. One enters the water at A and is reflected from the mirror at B. This ray is perpendicular to the water line and mirror. The second ray leaves the lightbulb at the angle θ, enters the water at C, where it is refracted. It is reflected from the mirror at D and leaves the water at E. At C the angle of incidence is θ and the angle of refraction is θ'. At D the angles of incidence and reflection are both θ'. At E the angle of incidence is θ' and the angle of refraction is θ. The dotted lines that meet at I represent extensions of the emerging rays. Light appears to come from I. We want to compute d_3.

Consideration of the triangle OBE tells us that the distance $d_2 + d_3$ is $L \tan(90° - \theta) = L/\tan\theta$, where L is the distance between A and E. Consideration of the triangle OBC tells us that the distance between A and C is $d_1 \tan\theta$ and consideration of the triangle CDE tells us that the distance between C and E is $2d_2 \tan\theta'$, so $L = d_1 \tan\theta + 2d_2 \tan\theta'$, $d_2 + d_3 = (d_1 \tan\theta + 2d_2 \tan\theta')/\tan\theta$, and

$$d_3 = \frac{d_1 \tan\theta + 2d_2 \tan\theta'}{\tan\theta} - d_2 .$$

Apply the law of refraction at point C: $\sin\theta = n \sin\theta'$, where n is the index of refraction of water. Since the angles θ and θ' are small we may approximate their sines by their tangents and write $\tan\theta = n \tan\theta'$. Us this to substitute for $\tan\theta$ in the expression for d_3 to obtain

$$d_3 = \frac{nd_1 + 2d_2}{n} - d_2 = \frac{(1.33)(250\,\text{cm}) + 2(200\,\text{cm})}{1.33} - 200\,\text{cm} = 350\,\text{cm}\,,$$

where the index of refraction of water was taken to be 1.33.

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(a) The radius of curvature r and focal length f are positive for a concave mirror and are related by $f = r/2$, so $r = 2(+12\,\text{cm}) = +24\,\text{cm}$.

(b) Since $(1/p) + (1/i) = 1/f$, where i is the image distance,

$$i = \frac{fp}{p - f} = \frac{(12\,\text{cm})(18\,\text{cm})}{18\,\text{cm} - 12\,\text{cm}} = 36\,\text{cm}\,.$$

(c) The magnification is $m = -i/p = -(36\,\text{cm})/(18\,\text{cm} = -2.0$.

(d) The value obtained for i is positive, so the image is real.

(e) The value obtained for the magnification is negative, so the image is inverted.

(f) Real images are formed by mirrors on the same side as the object. Since the image here is real it is on the same side of the mirror as the object.

<u>9</u>

(a) The radius of curvature r and focal length f are positive for a concave mirror and are related by $f = r/2$, so $r = 2(+18\,\text{cm}) = +36\,\text{cm}$.

(b) Since $(1/p) + (1/i) = 1/f$, where i is the image distance,

$$i = \frac{fp}{p - f} = \frac{(18\,\text{cm})(12\,\text{cm})}{12\,\text{cm} - 18\,\text{cm}} = -36\,\text{cm}\,.$$

(c) The magnification is $m = -i/p = -(-36\,\text{cm})/(12\,\text{cm} = 3.0$.

(d) The value obtained for i is negative, so the image is virtual.

(e) The value obtained for the magnification is positive, so the image is not inverted.

(f) Real images are formed by mirrors on the same side as the object and virtual images are formed on the opposite side. Since the image here is virtual it is on the opposite side of the mirror from the object.

<u>15</u>

(a) The radius of curvature r and focal length f are negative for a convex mirror and are related by $f = r/2$, so $r = 2(-10\,\text{cm}) = -20\,\text{cm}$.

(b) Since $(1/p) + (1/i) = 1/f$, where i is the image distance,

$$i = \frac{fp}{p - f} = \frac{(-10\,\text{cm})(8\,\text{cm})}{(8\,\text{cm}) - (-10\,\text{cm})} = -4.44\,\text{cm}\,.$$

(c) The magnification is $m = -i/p = -(-4.44\,\text{cm})/(8\,\text{cm} = +0.56$.

(d) The value obtained for i is negative, so the image is virtual.

(e) The value obtained for the magnification is positive, so the image is not inverted.

(f) Real images are formed by mirrors on the same side as the object and virtual images are formed on the opposite side. Since the image here is virtual it is on the opposite side of the mirror from the object

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Since the mirror is convex the radius of curvature is negative. The focal length is $f = r/2 = (-40\,\text{cm})/2 = -20\,\text{cm}$.

Since $(1/p) + (1/i) = (1/f)$,

$$p = \frac{if}{i - f}\,.$$

This yields $p = +5.0$ cm if $i = -4.0$ cm and $p = -3.3$ cm if $i = -4.0$ cm. Since p must be positive we select $i = -4.0$ cm and take p to be $+5.0$ cm.

The magnification is $m = -i/p = -(-4.0\,\text{cm})/(5.0\,\text{cm}) = +0.80$. Since the image distance is negative the image is virtual and on the opposite side of the mirror from the object. Since the magnification is positive the image is not inverted.

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Since the magnification m is $m = -i/p$, where p is the object distance and i is the image distance, $i = -mp$. Use this to substitute for i in $(1/p) + (1/i) = (1/f)$, where f is the focal length. The solve for p. The result is

$$p = f\left(1 - \frac{1}{m}\right) = (\pm 30\,\text{cm})\left(1 - \frac{1}{0.20}\right) = \pm 120\,\text{cm}\,.$$

Since p must be positive we must use the lower sign. Thus the focal length is -30 cm and the radius of curvature is $r = 2f == 60$ cm. Since the focal length and radius of curvature are negative the mirror is convex.

The object distance is 1.2 m and the image distance is $i = -mp = -(0.20)(120\,\text{cm}) = -24$ cm.

Since the image distance is negative the image is virtual and on the opposite side of the mirror from the object. Since the magnification is positive the image is not inverted.

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Solve

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}$$

for r. the result is

$$r = \frac{ip(n_2 - n_1)}{n_1 i + n_2 p} = \frac{(-13\,\text{cm})(+10\,\text{cm})}{((1.0)(-13\,\text{cm}) + (1.5)(+10\,\text{cm})} = -33\,\text{cm}\,.$$

Since the image distance is negative the image is virtual and appears on the same side of the surface as the object.

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Solve

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}$$

for r. the result is

$$i = \frac{n_2 r p}{(n_2 - n_1)p - n_1 r} = \frac{(1.0)(+30\,\text{cm})(+70\,\text{cm})}{(1.0 - 1.5)(+70\,\text{cm}) - (1.5)(+30\,\text{cm})} = -26\,\text{cm}\,.$$

Since the image distance is negative the image is virtual and appears on the same side of the surface as the object.

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Use the lens maker's equation, Eq. 34–10:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right),$$

where f is the focal length, n is the index of refraction, r_1 is the radius of curvature of the first surface encountered by the light and r_2 is the radius of curvature of the second surface. Since one surface has twice the radius of the other and since one surface is convex to the incoming light while the other is concave, set $r_2 = -2r_1$ to obtain

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{2r_1}\right) = \frac{3(n-1)}{2r_1}.$$

Solve for r_1:

$$r_1 = \frac{3(n-1)f}{2} = \frac{3(1.5-1)(60\,\text{mm})}{2} = 45\,\text{mm}.$$

The radii are 45 mm and 90 mm.

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The object distance p and image distance i obey $(1/p) + (1/i) = (1/f)$, where f is the focal length. In addition, $p + i = L$, where L (= 44 cm) is the distance from the slide to the screen. Use $i = L - p$ to substitute for i in the first equation and obtain $p^2 - pL + Lf = 0$. The solution is

$$p = \frac{L \pm \sqrt{L^2 - 4Lf}}{2} = \frac{(44\,\text{cm}) \pm \sqrt{4(44\,\text{cm})(11\,\text{cm})}}{2} = 22\,\text{cm}.$$

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The lens is diverging, so the focal length is negative. Solve $(1/p) + (1/i) = (1/f)$ for i. The result is

$$i = \frac{pf}{p-f} = \frac{(+8.0\,\text{cm})(-12\,\text{cm})}{(8.0\,\text{cm}) - (-12\,\text{cm})} = -4.8\,\text{cm}.$$

The magnification is $m = -i/p = -(-4.8\,\text{cm})/(+8.0\,\text{cm}) = 0.60$. Since the image distance is negative the image is virtual and appears on the same side of the lens as the object. Since the magnification is positive the image is not inverted.

55

The lens is converging, so the focal length is positive. Solve $(1/p) + (1/i) = (1/f)$ for i. The result is

$$i = \frac{pf}{p-f} = \frac{(+45\,\text{cm})(+20\,\text{cm})}{(45\,\text{cm}) - (+20\,\text{cm})} = +36\,\text{cm}.$$

The magnification is $m = -i/p = -(36\,\text{cm})/(45\,\text{cm}) = -0.80$. Since the image distance is positive the image is real and appears on the opposite side of the lens from the object. Since the magnification is negative the image is inverted.

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The focal length is

$$f = \frac{r_1 r_2}{(n-1)(r_2 - r_1)} = \frac{(+30\,\text{cm})(-42\,\text{cm})}{(1.55-1)[(-42\,\text{cm}) - (+30\,\text{cm})]} = +31.8\,\text{cm}\,.$$

Solve $(1/p) + (1/i) = (1/f)$ for i. the result is

$$i = \frac{pf}{p-f} = \frac{(+75\,\text{cm})(+31.8\,\text{cm})}{(+75\,\text{cm}) - (+31.8\,\text{cm})} = 55\,\text{cm}\,.$$

The magnification is $m = -i/p = -(55\,\text{cm})/(75\,\text{cm}) = -0.73$.

Since the image distance is positive the image is real and on the opposite side of the lends from the object. Since the magnification is negative the image is inverted.

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Since $m = -i/p$, $i = -mp = -(+1.25)(+16\,\text{cm}) = -20\,\text{cm}$. Solve $(1/p) + (1/i) = (1/f)$ for f. The result is

$$f = \frac{pi}{p+i} = \frac{(+16\,\text{cm})(-20\,\text{cm})}{(+16\,\text{cm}) + (-20\,\text{cm})} = +80\,\text{cm}\,.$$

Since f is positive the lens is a converging lens. Since the image distance is negative the image is virtual and appears on the same side of the lens as the object. Since the magnification is positive the image is not inverted.

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The image is on the same side of the lens as the object. This means that the image is virtual and the image distance is negative. Solve $(1/p) + (1/i) = (1/f)$ for i. The result is

$$i = \frac{pf}{p-f}\,.$$

and the magnification is

$$m = -\frac{i}{p} = -\frac{f}{p-f}\,.$$

Since the magnification is less than 1.0, f must be negative and the lens must be a diverging lens. The image distance is

$$i = \frac{(+5.0\,\text{cm})(-10\,\text{cm})}{(5.0\,\text{cm}) - (-10\,\text{cm})} = -3.3\,\text{cm}\,.$$

and the magnification is $m = -i/p = -(-3.3\,\text{cm})/(5.0\,\text{cm}) = 0.66\,\text{cm}$.

Since the magnification is positive the image is not inverted.

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Lens 1 is converging and so has a positive focal length. Solve $(1/p_1) + (1/i_1) = (1/f_1)$ for the image distance i_1 associated with the image produced by this lens. The result is

$$i_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(20\,\text{cm})(+9.0\,\text{cm})}{(20\,\text{cm}) - (9.0\,\text{cm})} = 16.4\,\text{cm}\,.$$

This image is the object for lens 2. The object distance is $d - p_2 = (8.0\,\text{cm}) - (16.4\,\text{cm}) = -8.4\,\text{cm}$. The negative sign indicates that the image is behind the second lens. The lens equation is still valid. The second lens has a positive focal length and the image distance for the image it forms is

$$i_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(-8.4\,\text{cm})(5.0\,\text{cm})}{(-8.4\,\text{cm}) - (5.0\,\text{cm})} = +3.1\,\text{cm}\,.$$

The overall magnification is the product of the individual magnifications:

$$m = m_1 m_2 = \left(-\frac{i_1}{p_1}\right)\left(-\frac{i_2}{p_2}\right) = \left(-\frac{16.4\,\text{cm}}{20\,\text{cm}}\right)\left(-\frac{3.1\,\text{cm}}{-8.4\,\text{cm}}\right) = -0.30\,.$$

Since the final image distance is positive the final image is real and on the opposite side of lens 2 from the object. Since the magnification is negative the image is inverted.

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(a) If L is the distance between the lenses, then according to Fig. 34–20, the tube length is $s = L - f_{\text{ob}} - f_{\text{ey}} = 25.0\,\text{cm} - 4.00\,\text{cm} - 8.00\,\text{cm} = 13.0\,\text{cm}$.

(b) Solve $(1/p) + (1/i) = (1/f_{\text{ob}})$ for p. The image distance is $i = f_{\text{ob}} + s = 4.00\,\text{cm} + 13.0\,\text{cm} = 17.0\,\text{cm}$, so

$$p = \frac{i f_{\text{ob}}}{i - f_{\text{ob}}} = \frac{(17.0\,\text{cm})(4.00\,\text{cm})}{17.0\,\text{cm} - 4.00\,\text{cm}} = 5.23\,\text{cm}\,.$$

(c) The magnification of the objective is

$$m = -\frac{i}{p} = -\frac{17.0\,\text{cm}}{5.23\,\text{cm}} = -3.25\,.$$

(d) The angular magnification of the eyepiece is

$$m_\theta = \frac{25\,\text{cm}}{f_{\text{ey}}} = \frac{25\,\text{cm}}{8.00\,\text{cm}} = 3.13\,.$$

(e) The overall magnification of the microscope is

$$M = m m_\theta = (-3.25)(3.13) = -10.2\,.$$

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(a) When the eye is relaxed, its lens focuses far-away objects on the retina, a distance i behind the lens. Set $p = \infty$ in the thin lens equation to obtain $1/i = 1/f$, where f is the focal length of the relaxed effective lens. Thus $i = f = 2.50\,\text{cm}$. When the eye focuses on closer objects, the image distance i remains the same but the object distance and focal length change. If p is the new object distance and f' is the new focal length, then

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f'} \,.$$

Substitute $i = f$ and solve for f'. You should obtain

$$f' = \frac{pf}{f+p} = \frac{(40.0\,\text{cm})(2.50\,\text{cm})}{40.0\,\text{cm} + 2.50\,\text{cm}} = 2.35\,\text{cm}\,.$$

(b) Consider the lensmaker's equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \,,$$

where r_1 and r_2 are the radii of curvature of the two surfaces of the lens and n is the index of refraction of the lens material. For the lens pictured in Fig. 34–46, r_1 and r_2 have about the same magnitude, r_1 is positive, and r_2 is negative. Since the focal length decreases, the combination $(1/r_1) - (1/r_2)$ must increase. This can be accomplished by decreasing the magnitudes of either or both radii.

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For a thin lens, $(1/p) + (1/i) = (1/f)$, where p is the object distance, i is the image distance, and f is the focal length. Solve for i:

$$i = \frac{fp}{p-f} \,.$$

Let $p = f + x$, where x is positive if the object is outside the focal point and negative if it is inside. Then

$$i = \frac{f(f+x)}{x} \,.$$

Now let $i = f + x'$, where x' is positive if the image is outside the focal point and negative if it is inside. Then

$$x' = i - f = \frac{f(f+x)}{x} - f = \frac{f^2}{x}$$

and $xx' = f^2$.

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Place an object far away from the composite lens and find the image distance i. Since the image is at a focal point, $i = f$, the effective focal length of the composite. The final image is produced by two lenses, with the image of the first lens being the object for the second. For the first lens, $(1/p_1) + (1/i_1) = (1/f_1)$, where f_1 is the focal length of this lens and i_1 is the image distance for the image it forms. Since $p_1 = \infty$, $i_1 = f_1$.

The thin lens equation, applied to the second lens, is $(1/p_2) + (1/i_2) = (1/f_2)$, where p_2 is the object distance, i_2 is the image distance, and f_2 is the focal length. If the thicknesses of the lenses can be ignored, the object distance for the second lens is $p_2 = -i_1$. The negative sign must be used since the image formed by the first lens is beyond the second lens if i_1 is positive. This means the object for the second lens is virtual and the object distance is negative. If i_1 is negative, the image formed by the first lens is in front of the second lens and p_2 is positive. In the thin lens equation, replace p_2 with $-f_1$ and i_2 with f to obtain

$$-\frac{1}{f_1} + \frac{1}{f} = \frac{1}{f_2}\,.$$

The solution for f is

$$f = \frac{f_1 f_2}{f_1 + f_2}\,.$$

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(a) and (b) Since the height of the image is twice the height of the fly and since the fly and its image have the same orientation the magnification of the lens is $m = +2.0$. Since $m = -i/p$, where p is the object distance and i is the image distance, $i = -2p$. Now $|p+i| = d$, so $|-p| = d$ and $p = d = 20\,\text{cm}$. The image distance is $-40\,\text{cm}$.

Solve $(1/p) + (1/i) = (1/f)$ for f. the result is

$$f = \frac{pi}{p+i} = \frac{(20\,\text{cm})(-40\,\text{cm})}{(20\,\text{cm}) + (-40\,\text{cm})} = +40\,\text{cm}\,.$$

(c) and (d) Now $m = +0.5$ and $i = -0.5p$. Since $|p+i| = d$, $0.5p = d$ and $p = 2d = 40\,\text{cm}$. The image distance is $-20\,\text{cm}$ and the focal length is

$$f = \frac{pi}{p+i} = \frac{(40\,\text{cm})(-20\,\text{cm})}{(40\,\text{cm}) + (-20\,\text{cm})} = -40\,\text{cm}\,.$$