Chapter 33

5

If f is the frequency and λ is the wavelength of an electromagnetic wave, then $f\lambda = c$. The frequency is the same as the frequency of oscillation of the current in the LC circuit of the generator. That is, $f = 1/2\pi\sqrt{LC}$, where C is the capacitance and L is the inductance. Thus

$$\frac{\lambda}{2\pi\sqrt{LC}} = c\,.$$

The solution for L is

$$L = \frac{\lambda^2}{4\pi^2 C c^2} = \frac{(550 \times 10^{-9}\,\text{m})^2}{4\pi^2(17 \times 10^{-12}\,\text{F})(3.00 \times 10^8\,\text{m/s})^2} = 5.00 \times 10^{-21}\,\text{H}\,.$$

This is exceedingly small.

21

The plasma completely reflects all the energy incident on it, so the radiation pressure is given by $p_r = 2I/c$, where I is the intensity. The intensity is $I = P/A$, where P is the power and A is the area intercepted by the radiation. Thus

$$p_r = \frac{2P}{Ac} = \frac{2(1.5 \times 10^9\,\text{W})}{(1.00 \times 10^{-6}\,\text{m}^2)(3.00 \times 10^8\,\text{m/s})} = 1.0 \times 10^7\,\text{Pa} = 10\,\text{MPa}\,.$$

23

Let f be the fraction of the incident beam intensity that is reflected. The fraction absorbed is $1 - f$. The reflected portion exerts a radiation pressure of $p_r = (2fI_0)/c$ and the absorbed portion exerts a radiation pressure of $p_a = (1 - f)I_0/c$, where I_0 is the incident intensity. The factor 2 enters the first expression because the momentum of the reflected portion is reversed. The total radiation pressure is the sum of the two contributions:

$$p_{\text{total}} = p_r + p_a = \frac{2fI_0 + (1 - f)I_0}{c} = \frac{(1 + f)I_0}{c}\,.$$

To relate the intensity and energy density, consider a tube with length ℓ and cross-sectional area A, lying with its axis along the propagation direction of an electromagnetic wave. The electromagnetic energy inside is $U = uA\ell$, where u is the energy density. All this energy will pass through the end in time $t = \ell/c$ so the intensity is

$$I = \frac{U}{At} = \frac{uA\ell c}{A\ell} = uc\,.$$

Thus $u = I/c$. The intensity and energy density are inherently positive, regardless of the propagation direction.

For the partially reflected and partially absorbed wave, the intensity just outside the surface is $I = I_0 + fI_0 = (1+f)I_0$, where the first term is associated with the incident beam and the second is associated with the reflected beam. The energy density is, therefore,

$$u = \frac{I}{c} = \frac{(1+f)I_0}{c},$$

the same as radiation pressure.

<u>**25**</u>

(a) Since $c = \lambda f$, where λ is the wavelength and f is the frequency of the wave,

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8\,\text{m/s}}{3.0\,\text{m}} = 1.0 \times 10^8\,\text{Hz}.$$

(b) The angular frequency is

$$\omega = 2\pi f = 2\pi(1.0 \times 10^8\,\text{Hz}) = 6.3 \times 10^8\,\text{rad/s}.$$

(c) The angular wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.0\,\text{m}} = 2.1\,\text{rad/m}.$$

(d) The magnetic field amplitude is

$$B_m = \frac{E_m}{c} = \frac{300\,\text{V/m}}{3.00 \times 10^8\,\text{m/s}} = 1.00 \times 10^{-6}\,\text{T}.$$

(e) $\vec{B}$ must be in the positive z direction when $\vec{E}$ is in the positive y direction in order for $\vec{E} \times \vec{B}$ to be in the positive x direction (the direction of propagation).

(f) The time-averaged rate of energy flow or intensity of the wave is

$$I = \frac{E_m^2}{2\mu_0 c} = \frac{(300\,\text{V/m})^2}{2(4\pi \times 10^{-7}\,\text{H/m})(3.00 \times 10^8\,\text{m/s})} = 1.2 \times 10^2\,\text{W/m}^2.$$

(g) Since the sheet is perfectly absorbing, the rate per unit area with which momentum is delivered to it is I/c, so

$$\frac{dp}{dt} = \frac{IA}{c} = \frac{(119\,\text{W/m}^2)(2.0\,\text{m}^2)}{3.00 \times 10^8\,\text{m/s}} = 8.0 \times 10^{-7}\,\text{N}.$$

(h) The radiation pressure is

$$p_r = \frac{dp/dt}{A} = \frac{8.0 \times 10^{-7}\,\text{N}}{2.0\,\text{m}^2} = 4.0 \times 10^{-7}\,\text{Pa}.$$

<u>**27**</u>

If the beam carries energy U away from the spaceship, then it also carries momentum $p = U/c$ away. Since the total momentum of the spaceship and light is conserved, this is the magnitude of the momentum acquired by the spaceship. If P is the power of the laser, then the energy carried away in time t is $U = Pt$. Thus $p = Pt/c$ and, if m is mass of the spaceship, its speed is

$$v = \frac{p}{m} = \frac{Pt}{mc} = \frac{(10 \times 10^3\,\text{W})(1\,d)(8.64 \times 10^4\,\text{s/d})}{(1.5 \times 10^3\,\text{kg})(3.00 \times 10^8\,\text{m/s})} = 1.9 \times 10^{-3}\,\text{m/s} = 1.9\,\text{mm/s}\,.$$

35

Let I_0 be in the intensity of the unpolarized light that is incident on the first polarizing sheet. Then the transmitted intensity is $I_1 = \frac{1}{2}I_0$ and the direction of polarization of the transmitted light is θ_1 (= 40°) counterclockwise from the y axis in the diagram.

The polarizing direction of the second sheet is θ_2 (= 20°) clockwise from the y axis so the angle between the direction of polarization of the light that is incident on that sheet and the polarizing direction of the of the sheet is 40° + 20° = 60°. The transmitted intensity is

$$I_2 = I_1 \cos^2 60^\circ = \frac{1}{2} I_0 \cos^2 60^\circ$$

and the direction of polarization of the transmitted light is 20° clockwise from the y axis.

The polarizing direction of the third sheet is θ_3 (= 40°) counterclockwise from the y axis so the angle between the direction of polarization of the light incident on that sheet and the polarizing direction of the sheet is 20° + 40° = 60°. The transmitted intensity is

$$I_3 = I_2 \cos^2 60^\circ = \frac{1}{2} I_0 \cos^4 60^\circ = 3.1 \times 10^{-2}\,.$$

3.1% of the light's initial intensity is transmitted.

43

(a) The rotation cannot be done with a single sheet. If a sheet is placed with its polarizing direction at an angle of 90° to the direction of polarization of the incident radiation, no radiation is transmitted.

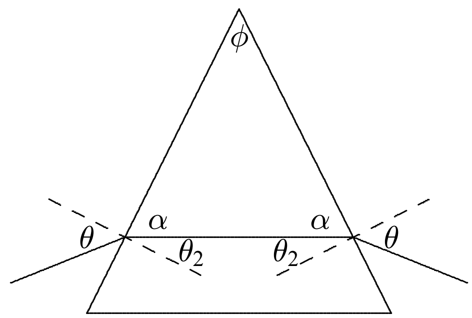
It can be done with two sheets. Place the first sheet with its polarizing direction at some angle θ, between 0 and 90°, to the direction of polarization of the incident radiation. Place the second sheet with its polarizing direction at 90° to the polarization direction of the incident radiation. The transmitted radiation is then polarized at 90° to the incident polarization direction. The intensity is $I_0 \cos^2\theta \cos^2(90^\circ - \theta) = I_0 \cos^2\theta \sin^2\theta$, where I_0 is the incident radiation. If θ is not 0 or 90°, the transmitted intensity is not zero.

(b) Consider n sheets, with the polarizing direction of the first sheet making an angle of $\theta = 90^\circ/n$ with the direction of polarization of the incident radiation and with the polarizing direction of each successive sheet rotated $90^\circ/n$ in the same direction from the polarizing direction of the previous sheet. The transmitted radiation is polarized with its direction of polarization making an angle of 90° with the direction of polarization of the incident radiation. The intensity is $I = I_0 \cos^{2n}(90^\circ/n)$. You want the smallest integer value of n for which this is greater than $0.60 I_0$.

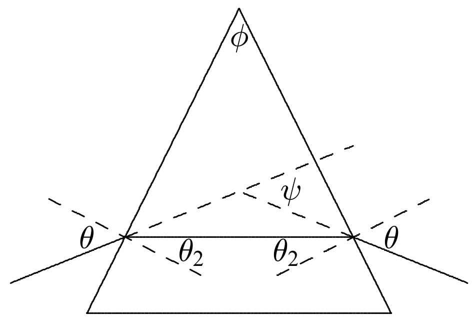
Start with $n = 2$ and calculate $\cos^{2n}(90°/n)$. If the result is greater than 0.60, you have obtained the solution. If it is less, increase n by 1 and try again. Repeat this process, increasing n by 1 each time, until you have a value for which $\cos^{2n}(90°/n)$ is greater than 0.60. The first one will be $n = 5$.

55

Look at the diagram on the right. The two angles labeled α have the same value. θ_2 is the angle of refraction. Because the dotted lines are perpendicular to the prism surface $\theta_2 + \alpha = 90°$ and $\alpha = 90° - \theta_2$. Because the interior angles of a triangle sum to 180°, $180° - 2\theta_2 + \phi = 180°$ and $\theta_2 = \phi/2$.



Now look at the next diagram and consider the triangle formed by the two normals and the ray in the interior. The two equal interior angles each have the value $\theta - \theta_2$. Because the exterior angle of a triangle is equal to the sum of the two opposite interior angles, $\psi = 2(\theta - \theta_2)$ and $\theta = \theta_2 + \psi/2$. Upon substitution for θ_2 this becomes $\theta = (\phi + \psi)/2$.



According to the law of refraction the index of refraction of the prism material is

$$n = \frac{\sin\theta}{\sin\theta_2} = \frac{\sin(\phi + \psi)/2}{\sin\phi/2} .$$

65

(a) No refraction occurs at the surface ab, so the angle of incidence at surface ac is $90° - \phi$. For total internal reflection at the second surface, $n_g \sin(90° - \phi)$ must be greater than n_a. Here n_g is the index of refraction for the glass and n_a is the index of refraction for air. Since $\sin(90° - \phi) = \cos\phi$, you want the largest value of ϕ for which $n_g \cos\phi \geq n_a$. Recall that $\cos\phi$ decreases as ϕ increases from zero. When ϕ has the largest value for which total internal reflection occurs, then $n_g \cos\phi = n_a$, or

$$\phi = \cos^{-1}\left(\frac{n_a}{n_g}\right) = \cos^{-1}\left(\frac{1}{1.52}\right) = 48.9° .$$

The index of refraction for air was taken to be unity.

(b) Replace the air with water. If n_w (= 1.33) is the index of refraction for water, then the largest value of ϕ for which total internal reflection occurs is

$$\phi = \cos^{-1}\left(\frac{n_w}{n_g}\right) = \cos^{-1}\left(\frac{1.33}{1.52}\right) = 29.0° .$$

69

The angle of incidence θ_B for which reflected light is fully polarized is given by Eq. 33–49 of the text. If n_1 is the index of refraction for the medium of incidence and n_2 is the index of refraction for the second medium, then $\theta_B = \tan^{-1}(n_2/n_1) = \tan^{-1}(1.53/1.33) = 63.8°$.

73

Let θ_1 (= 45°) be the angle of incidence at the first surface and θ_2 be the angle of refraction there. Let θ_3 be the angle of incidence at the second surface. The condition for total internal reflection at the second surface is $n \sin\theta_3 \geq 1$. You want to find the smallest value of the index of refraction n for which this inequality holds.

The law of refraction, applied to the first surface, yields $n \sin\theta_2 = \sin\theta_1$. Consideration of the triangle formed by the surface of the slab and the ray in the slab tells us that $\theta_3 = 90° - \theta_2$. Thus the condition for total internal reflection becomes $1 \leq n \sin(90° - \theta_2) = n \cos\theta_2$. Square this equation and use $\sin^2\theta_2 + \cos^2\theta_2 = 1$ to obtain $1 \leq n^2(1 - \sin^2\theta_2)$. Now substitute $\sin\theta_2 = (1/n)\sin\theta_1$ to obtain

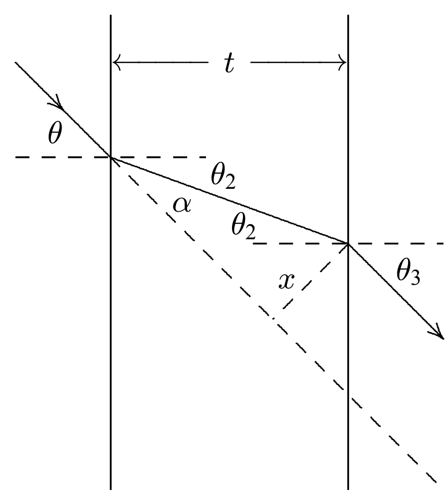
$$1 \leq n^2\left(1 - \frac{\sin^2\theta_1}{n^2}\right) = n^2 - \sin^2\theta_1 .$$

The largest value of n for which this equation is true is the value for which $1 = n^2 - \sin^2\theta_1$. Solve for n:

$$n = \sqrt{1 + \sin^2\theta_1} = \sqrt{1 + \sin^2 45°} = 1.22 .$$

75

Let θ be the angle of incidence and θ_2 be the angle of refraction at the left face of the plate. Let n be the index of refraction of the glass. Then, the law of refraction yields $\sin\theta = n\sin\theta_2$. The angle of incidence at the right face is also θ_2. If θ_3 is the angle of emergence there, then $n\sin\theta_2 = \sin\theta_3$. Thus $\sin\theta_3 = \sin\theta$ and $\theta_3 = \theta$. The emerging ray is parallel to the incident ray.



You wish to derive an expression for x in terms of θ. If D is the length of the ray in the glass, then $D\cos\theta_2 = t$ and $D = t/\cos\theta_2$. The angle α in the diagram equals $\theta - \theta_2$ and $x = D\sin\alpha = D\sin(\theta - \theta_2)$. Thus

$$x = \frac{t\sin(\theta - \theta_2)}{\cos\theta_2} .$$

If all the angles θ, θ_2, θ_3, and $\theta - \theta_2$ are small and measured in radians, then $\sin\theta \approx \theta$, $\sin\theta_2 \approx \theta_2$, $\sin(\theta - \theta_2) \approx \theta - \theta_2$, and $\cos\theta_2 \approx 1$. Thus $x \approx t(\theta - \theta_2)$. The law of refraction applied to the point of incidence at the left face of the plate is now $\theta \approx n\theta_2$, so $\theta_2 \approx \theta/n$ and

$$x \approx t\left(\theta - \frac{\theta}{n}\right) = \frac{(n-1)t\theta}{n}\,.$$

<u>77</u>

The time for light to travel a distance d in free space is $t = d/c$, where c is the speed of light (3.00×10^8 m/s).

(a) Take d to be 150 km = 150×10^3 m. Then,

$$t = \frac{d}{c} = \frac{150 \times 10^3\ \text{m}}{3.00 \times 10^8\ \text{m/s}} = 5.00 \times 10^{-4}\ \text{s}\,.$$

(b) At full moon, the Moon and Sun are on opposite sides of Earth, so the distance traveled by the light is $d = (1.5 \times 10^8\ \text{km}) + 2(3.8 \times 10^5\ \text{km}) = 1.51 \times 10^8\ \text{km} = 1.51 \times 10^{11}\ \text{m}$. The time taken by light to travel this distance is

$$t = \frac{d}{c} = \frac{1.51 \times 10^{11}\ \text{m}}{3.00 \times 10^8\ \text{m/s}} = 500\ \text{s} = 8.4\ \text{min}\,.$$

The distances are given in the problem.

(c) Take d to be $2(1.3 \times 10^9\ \text{km}) = 2.6 \times 10^{12}$ m. Then,

$$t = \frac{d}{c} = \frac{2.6 \times 10^{12}\ \text{m}}{3.00 \times 10^8\ \text{m/s}} = 8.7 \times 10^3\ \text{s} = 2.4\ \text{h}\,.$$

(d) Take d to be 6500 ly and the speed of light to be 1.00 ly/y. Then,

$$t = \frac{d}{c} = \frac{6500\ \text{ly}}{1.00\ \text{ly/y}} = 6500\ \text{y}\,.$$

The explosion took place in the year 1054 − 6500 = −5446 or B.C. 5446.

<u>79</u>

(a) The amplitude of the magnetic field is $B = E/c = (5.00\ \text{V/m})/(3.00 \times 10^8\ \text{m/s}) = 1.67 \times 10^{-8}$ T. According to the argument of the trigonometric function in the expression for the electric field, the wave is moving in the negative z direction and the electric field is parallel to the y axis. In order for $\vec{E} \times \vec{B}$ to be in the negative z direction, $\vec{B}$ must be in the positive x direction when $\vec{E}$ is in the positive y direction. Thus

$$B_x = (1.67 \times 10^{-8}\ \text{T})\sin[(1.00 \times 10^6\ \text{m}^{-1})z + \omega t]$$

is the only nonvanishing component of the magnetic field.

The angular wave number is $k = 1.00 \times 10^6\ \text{m}^{-1}$ so the angular frequency is $\omega = kc = (1.00 \times 10^6\ \text{m}^{-1})(3.00 \times 10^8\ \text{m/s}) = 3.00 \times 10^{14}\ \text{s}^{-1}$ and

$$B_x = (1.67 \times 10^{-8}\ \text{T})\sin[(1.00 \times 10^6\ \text{m}^{-1})z + (3.00 \times 10^{14}\ \text{s}^{-1})t]\,.$$

(b) The wavelength is $\lambda = 2\pi/k = 2\pi/(1.00 \times 10^6\ \text{m}^{-1}) = 6.28 \times 10^{-6}$ m.

(c) The period is $T = 2\pi/\omega = 2\pi/(3.00 \times 10^{14}\,\text{s}^{-1}) = 2.09 \times 10^{-14}\,\text{s}$.

(d) The intensity of this wave is $I = E_m^2/2\mu_0 c = (5.00\,\text{V/m})^2/2(4\pi\times10^{-7}\,\text{H/m})(3.00\times10^8\,\text{m/s} = 0.0332\,\text{W/m}^2$. (f) A wavelength of $6.28 \times 10^{-6}\,\text{m}$ places this wave in the infrared portion of the electromagnetic spectrum. See Fig. 33-1.

83

(a) The power is the same through any hemisphere centered at the source. The area of a hemisphere of radius r is $A = 2\pi r^2$. In this case r is the distance from the source to the aircraft. Thus the intensity at the aircraft is $I = P/A = P/2\pi r^2 = (180 \times 10^3\,\text{W})/2\pi(90 \times 10^3\,\text{m})^2 = 3.5 \times 10^{-6}\,\text{W/m}^2$.

(b) The power of the reflection is the product of the intensity at the aircraft and the cross section of the aircraft: $P_r = (3.5 \times 10^{-6}\,\text{W/m}^2)(0.22\,\text{m}^2) = 7.8 \times 10^{-7}\,\text{W}$.

(c) The intensity at the detector is $P_r/2\pi r^2 = (7.8 \times 10^{-7}\,\text{W})/2\pi(90 \times 10^3\,\text{m})^2 = 1.5 \times 10^{-17}\,\text{W/m}^2$.

(d) Since the intensity is given by $I = E_m^2/2\mu_0 c$,

$$E_m = \sqrt{2\mu_0 c I} = \sqrt{2(4\pi \times 10^{-7}\,\text{H/m})(3.00 \times 10^8\,\text{m/s})(1.5 \times 10^{-17}\,\text{W/m}^2)} = 1.1 \times 10^{-7}\,\text{V/m}\,.$$

(e) The rms value of the magnetic field is $B_{\text{rms}} = E_m/\sqrt{2}c = (1.1 \times 10^{-7}\,\text{V/m})/(\sqrt{2})(3.00 \times 10^8\,\text{m/s}) = 2.5 \times 10^{-16}\,\text{T}$.

91

The critical angle for total internal reflection is given by $\theta_c = \sin^{-1}(1/n)$. For $n = 1.456$ this angle is $\theta_c = 43.38°$ and for $n = 1.470$ it is $\theta_c = 42.86°$.

(a) An incidence angle of 42.00° is less than the critical angle for both red and blue light. The refracted light is white.

(b) An incidence angle of 43.10° is less than the critical angle for red light and greater than the critical angle for blue light. Red light is refracted but blue light is not. The refracted light is reddish.

(c) An incidence angle of 44.00° is greater than the critical angle for both red and blue light. Neither is refracted.

103

(a) Take the derivative of the functions given for E and B, then substitute them into

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2} \qquad \text{and} \qquad \frac{\partial^2 B}{\partial t^2} = c^2 \frac{\partial^2 B}{\partial x^2}\,.$$

The derivatives of E are $\partial^2 E/\partial t^2 = -\omega^2 E_m \sin(kx - \omega t)$ and $\partial^2 E/\partial x^2 = -k^2 E_m \sin(kx - \omega t)$, so the wave equation for the electric field yields $\omega^2 = c^2 k^2$. Since $\omega = ck$ the function satisfies the wave equation. Similarly, the derivatives of B are $\partial^2 B/\partial t^2 = -\omega^2 B_m \sin(kx - \omega t)$ and $\partial^2 B/\partial x^2 = -k^2 B_m \sin(kx - \omega t)$ and the wave equation for the magnetic field yields $\omega^2 = c^2 k^2$. Since $\omega = ck$ the function satisfies the wave equation.

(b) Let $u = kx \pm \omega t$ and consider f to be a function of u, which in turn is a function of x and t. Then the chain rule of the calculus gives

$$\frac{\partial^2 E}{\partial t^2} = \frac{d^2 f}{du^2}\left(\frac{\partial u}{\partial t}\right)^2 = \frac{d^2 f}{du^2}\omega^2$$

and

$$\frac{\partial^2 E}{\partial x^2} = \frac{d^2 f}{du^2}\left(\frac{\partial u}{\partial x}\right)^2 = \frac{d^2 f}{du^2}k^2 .$$

Substitution into the wave equation again yields $\omega^2 = c^2 k^2$, so the function obeys the wave equation. A similar analysis shows that the function for B also satisfies the wave equation.