Chapter 31

7

(a) The mass m corresponds to the inductance, so $m = 1.25\,\text{kg}$.

(b) The spring constant k corresponds to the reciprocal of the capacitance. Since the total energy is given by $U = Q^2/2C$, where Q is the maximum charge on the capacitor and C is the capacitance,

$$C = \frac{Q^2}{2U} = \frac{\left(175 \times 10^{-6}\,\text{C}\right)^2}{2(5.70 \times 10^{-6}\,\text{J})} = 2.69 \times 10^{-3}\,\text{F}$$

and

$$k = \frac{1}{2.69 \times 10^{-3}\,\text{m/N}} = 372\,\text{N/m}\,.$$

(c) The maximum displacement x_m corresponds to the maximum charge, so

$$x_m = 1.75 \times 10^{-4}\,\text{m}\,.$$

(d) The maximum speed v_m corresponds to the maximum current. The maximum current is

$$I = Q\omega = \frac{Q}{\sqrt{LC}} = \frac{175 \times 10^{-6}\,\text{C}}{\sqrt{(1.25\,\text{H})(2.69 \times 10^{-3}\,\text{F})}} = 3.02 \times 10^{-3}\,\text{A}\,.$$

Thus $v_m = 3.02 \times 10^{-3}\,\text{m/s}$.

15

(a) Since the frequency of oscillation f is related to the inductance L and capacitance C by $f = 1/2\pi\sqrt{LC}$, the smaller value of C gives the larger value of f. Hence, $f_{\text{max}} = 1/2\pi\sqrt{LC_{\text{min}}}$, $f_{\text{min}} = 1/2\pi\sqrt{LC_{\text{max}}}$, and

$$\frac{f_{\text{max}}}{f_{\text{min}}} = \frac{\sqrt{C_{\text{max}}}}{\sqrt{C_{\text{min}}}} = \frac{\sqrt{365\,\text{pF}}}{\sqrt{10\,\text{pF}}} = 6.0\,.$$

(b) You want to choose the additional capacitance C so the ratio of the frequencies is

$$r = \frac{1.60\,\text{MHz}}{0.54\,\text{MHz}} = 2.96\,.$$

Since the additional capacitor is in parallel with the tuning capacitor, its capacitance adds to that of the tuning capacitor. If C is in picofarads, then

$$\frac{\sqrt{C + 365\,\text{pF}}}{\sqrt{C + 10\,\text{pF}}} = 2.96\,.$$

The solution for C is

$$C = \frac{(365\,\text{pF}) - (2.96)^2(10\,\text{pF})}{(2.96)^2 - 1} = 36\,\text{pF}\,.$$

(c) Solve $f = 1/2\pi\sqrt{LC}$ for L. For the minimum frequency, $C = 365\,\text{pF} + 36\,\text{pF} = 401\,\text{pF}$ and $f = 0.54\,\text{MHz}$. Thus

$$L = \frac{1}{(2\pi)^2 C f^2} = \frac{1}{(2\pi)^2(401 \times 10^{-12}\,\text{F})(0.54 \times 10^6\,\text{Hz})^2} = 2.2 \times 10^{-4}\,\text{H}\,.$$

27

Let t be a time at which the capacitor is fully charged in some cycle and let $q_{\text{max 1}}$ be the charge on the capacitor then. The energy in the capacitor at that time is

$$U(t) = \frac{q_{\text{max 1}}^2}{2C} = \frac{Q^2}{2C}\, e^{-Rt/L}\,,$$

where

$$q_{\text{max 1}} = Q\, e^{-Rt/2L}$$

was used. Here Q is the charge at $t = 0$. One cycle later, the maximum charge is

$$q_{\text{max 2}} = Q\, e^{-R(t+T)/2L}$$

and the energy is

$$U(t+T) = \frac{q_{\text{max 2}}^2}{2C} = \frac{Q^2}{2C}\, e^{-R(t+T)/L}\,,$$

where T is the period of oscillation. The fractional loss in energy is

$$\frac{\Delta U}{U} = \frac{U(t) - U(t+T)}{U(t)} = \frac{e^{-Rt/L} - e^{-R(t+T)/L}}{e^{-Rt/L}} = 1 - e^{-RT/L}\,.$$

Assume that RT/L is small compared to 1 (the resistance is small) and use the Maclaurin series to expand the exponential. The first two terms are:

$$e^{-RT/L} \approx 1 - \frac{RT}{L}\,.$$

Replace T with $2\pi/\omega$, where ω is the angular frequency of oscillation. Thus

$$\frac{\Delta U}{U} \approx 1 - \left(1 - \frac{RT}{L}\right) = \frac{RT}{L} = \frac{2\pi R}{\omega L}\,.$$

33

(a) The generator emf is a maximum when $\sin(\omega_d t - \pi/4) = 1$ or $\omega_d t - \pi/4 = (\pi/2) \pm 2n\pi$, where n is an integer, including zero. The first time this occurs after $t = 0$ is when $\omega_d t - \pi/4 = \pi/2$ or

$$t = \frac{3\pi}{4\omega_d} = \frac{3\pi}{4(350\,\text{s}^{-1})} = 6.73 \times 10^{-3}\,\text{s}\,.$$

(b) The current is a maximum when $\sin(\omega_d t - 3\pi/4) = 1$, or $\omega_d t - 3\pi/4 = \pi/2 \pm 2n\pi$. The first time this occurs after $t = 0$ is when

$$t = \frac{5\pi}{4\omega_d} = \frac{5\pi}{4(350\,\text{s}^{-1})} = 1.12 \times 10^{-2}\,\text{s}\,.$$

(c) The current lags the inductor by $\pi/2$ rad, so the circuit element must be an inductor.

(d) The current amplitude I is related to the voltage amplitude V_L by $V_L = IX_L$, where X_L is the inductive reactance, given by $X_L = \omega_d L$. Furthermore, since there is only one element in the circuit, the amplitude of the potential difference across the element must be the same as the amplitude of the generator emf: $V_L = \mathcal{E}_m$. Thus $\mathcal{E}_m = I\omega_d L$ and

$$L = \frac{\mathcal{E}_m}{I\omega_d} = \frac{30.0\,\text{V}}{(620 \times 10^{-3}\,\text{A})(350\,\text{rad/s})} = 0.138\,\text{H}\,.$$

39

(a) The capacitive reactance is

$$X_C = \frac{1}{\omega_d C} = \frac{1}{2\pi f_d C} = \frac{1}{2\pi(60.0\,\text{Hz})(70.0 \times 10^{-6}\,\text{F})} = 37.9\,\Omega\,.$$

The inductive reactance is

$$X_L = \omega_d L = 1\pi f_d L = 2\pi(60.0\,\text{Hz})(230 \times 10^{-3}\,\text{H}) = 86.7\,\Omega\,.$$

The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\,\Omega)^2 + (37.9\,\Omega - 86.7\,\Omega)^2} = 206\,\Omega\,.$$

(b) The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7\,\Omega - 37.9\,\Omega}{200\,\Omega}\right) = 13.7^\circ\,.$$

(c) The current amplitude is

$$I = \frac{\mathcal{E}_m}{Z} = \frac{36.0\,\text{V}}{206\,\Omega} = 0.175\,\text{A}\,.$$

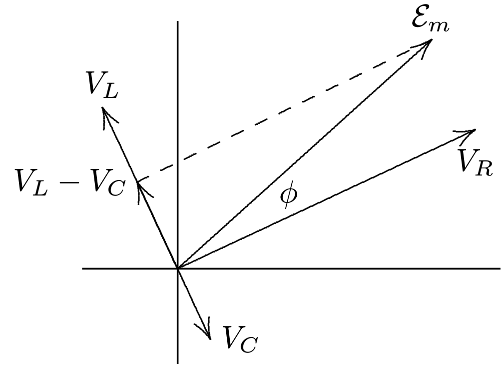
(d) The voltage amplitudes are

$$V_R = IR = (0.175\,\text{A})(200\,\Omega) = 35.0\,\text{V},$$

$$V_L = IX_L = (0.i75\,\text{A})(86.7\,\Omega) = 15.2\,\text{V},$$

and

$$V_C = IX_C = (0.175\,\text{A})(37.9\,\Omega) = 6,63\,\text{V}.$$



Note that $X_L > X_C$, so that $\mathcal{E}_m$ leads I. The phasor diagram is drawn to scale on the right.

45

(a) For a given amplitude $\mathcal{E}_m$ of the generator emf, the current amplitude is given by

$$I = \frac{\mathcal{E}_m}{Z} = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}\,,$$

where R is the resistance, L is the inductance, C is the capacitance, and ω_d is the angular frequency. To find the maximum, set the derivative with respect to ω_d equal to zero and solve for ω_d. The derivative is

$$\frac{dI}{d\omega_d} = -\mathcal{E}_m \left[R^2 + (\omega_d L - 1/\omega_d C)^2\right]^{-3/2} \left[\omega_d L - \frac{1}{\omega_d C}\right] \left[L + \frac{1}{\omega_d^2 C}\right] .$$

The only factor that can equal zero is $\omega_d L - (1/\omega_d C)$ and it does for $\omega_d = 1/\sqrt{LC}$. For the given circuit,

$$\omega_d = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})}} = 224\,\mathrm{rad/s} .$$

(b) For this value of the angular frequency, the impedance is $Z = R$ and the current amplitude is

$$I = \frac{\mathcal{E}_m}{R} = \frac{30.0\,\mathrm{V}}{5.00\,\Omega} = 6.00\,\mathrm{A} .$$

(c) and (d) You want to find the values of ω_d for which $I = \mathcal{E}_m/2R$. This means

$$\frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} = \frac{\mathcal{E}_m}{2R} .$$

Cancel the factors $\mathcal{E}_m$ that appear on both sides, square both sides, and set the reciprocals of the two sides equal to each other to obtain

$$R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2 = 4R^2 .$$

Thus

$$\left(\omega_d L - \frac{1}{\omega_d C}\right)^2 = 3R^2 .$$

Now take the square root of both sides and multiply by $\omega_d C$ to obtain

$$\omega_d^2 (LC) \pm \omega_d \left(\sqrt{3}CR\right) - 1 = 0 ,$$

where the symbol $\pm$ indicates the two possible signs for the square root. The last equation is a quadratic equation for ω_d. Its solutions are

$$\omega_d = \frac{\pm\sqrt{3}CR \pm \sqrt{3C^2R^2 + 4LC}}{2LC} .$$

You want the two positive solutions. The smaller of these is

$$\begin{aligned}
\omega_2 &= \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \\
&= \frac{-\sqrt{3}(20.0 \times 10^{-6}\,\mathrm{F})(5.00\,\Omega)}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&\qquad + \frac{\sqrt{3(20.0 \times 10^{-6}\,\mathrm{F})^2(5.00\,\Omega)^2 + 4(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})}}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&= 219\,\mathrm{rad/s}
\end{aligned}$$

and the larger is

$$\begin{aligned}\omega_1 &= \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \\ &= \frac{+\sqrt{3}(20.0 \times 10^{-6}\,\text{F})(5.00\,\Omega)}{2(1.00\,\text{H})(20.0 \times 10^{-6}\,\text{F})} \\ &\quad + \frac{\sqrt{3(20.0 \times 10^{-6}\,\text{F})^2(5.00\,\Omega)^2 + 4(1.00\,\text{H})(20.0 \times 10^{-6}\,\text{F})}}{2(1.00\,\text{H})(20.0 \times 10^{-6}\,\text{F})} \\ &= 228\,\text{rad/s}\,.\end{aligned}$$

(e) The fractional width is

$$\frac{\omega_1 - \omega_2}{\omega_0} = \frac{228\,\text{rad/s} - 219\,\text{rad/s}}{224\,\text{rad/s}} = 0.04\,.$$

<u>49</u>

Use the expressions found in Problem 31–45:

$$\omega_1 = \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC}$$

and

$$\omega_2 = \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC}\,.$$

Also use

$$\omega = \frac{1}{\sqrt{LC}}\,.$$

Thus

$$\frac{\Delta\omega_d}{\omega} = \frac{\omega_1 - \omega_2}{\omega} = \frac{2\sqrt{3}CR\sqrt{LC}}{2LC} = R\sqrt{\frac{3C}{L}}\,.$$

<u>55</u>

(a) The impedance is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}\,,$$

where R is the resistance, X_L is the inductive reactance, and X_C is the capacitive reactance. Thus

$$Z = \sqrt{(12.0\,\Omega)^2 + (1.30\,\Omega - 0)^2} = 12.1\,\Omega\,.$$

(b) The average rate at which energy is supplied to the air conditioner is given by

$$P_{\text{avg}} = \frac{\mathcal{E}_{\text{rms}}^2}{Z}\cos\phi\,,$$

where $\cos\phi$ is the power factor. Now

$$\cos\phi = \frac{R}{Z} = \frac{12\,\Omega}{12.1\,\Omega} = 0.992\,,$$

so

$$P_{\text{avg}} = \left[\frac{(120\text{ V})^2}{12.1\,\Omega}\right](0.992) = 1.18\times 10^3\text{ W}\,.$$

57

(a) The power factor is $\cos\phi$, where ϕ is the phase angle when the current is written $i = I\sin(\omega_d t - \phi)$. Thus $\phi = -42.0°$ and $\cos\phi = \cos(-42.0°) = 0.743$.

(b) Since $\phi < 0$, $\omega_d t - \phi > \omega_d t$ and the current leads the emf.

(c) The phase angle is given by $\tan\phi = (X_L - X_C)/R$, where X_L is the inductive reactance, X_C is the capacitive reactance, and R is the resistance. Now $\tan\phi = \tan(-42.0°) = -0.900$, a negative number. This means $X_L - X_C$ is negative, or $X_C > X_L$. The circuit in the box is predominantly capacitive.

(d) If the circuit is in resonance, X_L is the same as X_C, $\tan\phi$ is zero, and ϕ would be zero. Since ϕ is not zero, we conclude the circuit is not in resonance.

(e), (f), and (g) Since $\tan\phi$ is negative and finite, neither the capacitive reactance nor the resistance is zero. This means the box must contain a capacitor and a resistor. The inductive reactance may be zero, so there need not be an inductor. If there is an inductor, its reactance must be less than that of the capacitor at the operating frequency.

(h) The average power is

$$P_{\text{avg}} = \frac{1}{2}\mathcal{E}_m I\cos\phi = \frac{1}{2}(75.0\text{ V})(1.20\text{ A})(0.743) = 33.4\text{ W}\,.$$

(i) The answers above depend on the frequency only through the phase angle ϕ, which is given. If values are given for R, L, and C, then the value of the frequency would also be needed to compute the power factor.

63

(a) If N_p is the number of primary turns and N_s is the number of secondary turns, then

$$V_s = \frac{N_s}{N_p}V_p = \left(\frac{10}{500}\right)(120\text{ V}) = 2.4\text{ V}\,.$$

(b) and (c) The current in the secondary is given by Ohm's law:

$$I_s = \frac{V_s}{R_s} = \frac{2.4\text{ V}}{15\,\Omega} = 0.16\text{ A}\,.$$

The current in the primary is

$$I_p = \frac{N_s}{N_p}I_s = \left(\frac{10}{500}\right)(0.16\text{ A}) = 3.2\times 10^{-3}\text{ A}\,.$$

67

Use the trigonometric identity, found in Appendix E,

$$\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right),$$

where α and β are any two angles. Thus

$$V_1 - V_2 = A\sin(\omega_d t) - A\sin(\omega_d t - 120^\circ) = 2A\sin(120^\circ)\cos(\omega_d t - 60^\circ) = \sqrt{3}A\cos(\omega_d t - 60^\circ),$$

where $\sin(120^\circ) = \sqrt{3}/2$ was used. Similarly,

$$V_1 - V_3 = A\sin(\omega_d t) - A\sin(\omega_d t - 240^\circ) = 2A\sin(240^\circ)\cos(\omega_d t - 120^\circ) = -\sqrt{3}A\cos(\omega_d t - 120^\circ),$$

where $\sin(240^\circ) = -\sqrt{3}/2$ was used, and

$$\begin{aligned} V_2 - V_3 &= A\sin(\omega_d t - 120^\circ) - A\sin(\omega_d t - 240^\circ) = 2A\sin(120^\circ)\cos(\omega_d t - 180^\circ) \\ &= \sqrt{3}A\cos(\omega_d t - 180^\circ). \end{aligned}$$

All of these are sinusoidal functions of ω_d and all have amplitudes of $\sqrt{3}A$.

71

(a) Let V_C be the maximum potential difference across the capacitor, V_L be the maximum potential difference across the inductor, and V_R be the maximum potential difference across the resistor. Then the phase constant ϕ is

$$\tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = \tan^{-1}\left(\frac{2.00V_R - V_R}{V_R}\right) = \tan^{-1}(1.00) = 45.0^\circ.$$

(b) Since the maximum emf is related to the current amplitude by $\mathcal{E}_m = IZ$, where Z is the impedance and $R = Z\cos\phi$,

$$R = \frac{\mathcal{E}_m\cos\phi}{I} = \frac{(30.0\,\mathrm{V})\cos 45^\circ}{300\times10^{-3}\,\mathrm{A}} = 70.7\,\Omega.$$

73

(a) The frequency of oscillation of an LC circuit is $f = 1/2\pi\sqrt{LC}$, where L is the inductance and C is the capacitance. Thus

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2(10.4\times10^3\,\mathrm{Hz})^2(340\times10^{-6}\,\mathrm{F})} = 6.89\times10^{-7}\,\mathrm{H}.$$

(b) The total energy is $U = \frac{1}{2}LI^2$, where I is the current amplitude. Thus $U = \frac{1}{2}(6.89\times10^{-7}\,\mathrm{H})(7.20\times10^{-3}\,\mathrm{A})^2 = 1.79\times10^{-11}\,\mathrm{J}$.

(c) The total energy is also given by $U = Q^2/2C$, where Q is the charge amplitude. Thus $Q = \sqrt{2UC} = \sqrt{2(1.79 \times 10^{-11}\,\text{J})(340 \times 10^{-6}\,\text{F})} = 1.10 \times 10^{-7}\,\text{C}$.

83

(a) The total energy U of the circuit is the sum of the energy U_E stored in the capacitor and the energy U_B stored in the inductor at the same time. Since $U_B = 2.00U_E$, the total energy is $U = 3.00U_E$. Now $U = Q^2/2C$ and $U_E = q^2/2C$, where Q is the maximum charge, q is the charge when the magnetic energy is twice the electrical energy, and C is the capacitance. Thus $Q^2/2C = 3.00q^2/2C$ and $q = Q/\sqrt{3.00} = 0.577Q$.

(b) If the capacitor has maximum charge at time $t = 0$, then $q = Q\cos(\omega t)$, where ω is the angular frequency of oscillation. This means $\omega t = \cos^{-1}(0.577) = 0.964\,\text{rad}$. Since $\omega = 2\pi/T$, where T is the period,

$$t = \frac{0.964}{2\pi} T = 0.153T\,.$$

85

(a) The energy stored in a capacitor is given by $U_E = q^2/2C$, where q is the charge and C is the capacitance. Now q^2 is periodic with a period of $T/2$, where T is the period of the driving emf, so U_E has the same value at the beginning and end of each cycle. Actually U_E has the same value at the beginning and end of each half cycle.

(b) The energy stored in an inductor is given by $Li^2/2$, where i is the current and L is the inductance. The square of the current is periodic with a period of $T/2$, so it has the same value at the beginning and end of each cycle.

(c) The rate with which the driving emf device supplies energy is

$$P_{\mathcal{E}} = i\mathcal{E} = I\mathcal{E}_m \sin(\omega_d t)\sin(\omega_d t - \phi)\,,$$

where I is the current amplitude, $\mathcal{E}_m$ is the emf amplitude, ω is the angular frequency, and ϕ is a phase constant. The energy supplied over a cycle is

$$\begin{aligned} E_{\mathcal{E}} &= \int_0^T P_{\mathcal{E}}\,dt = I\mathcal{E}_m \int_0^T \sin(\omega_d t)\sin(\omega_d t - \phi)\,dt \\ &= I\mathcal{E}_m \int_0^T \sin(\omega_d t)\left[\sin(\omega_d t) cos(\phi) - \cos(\omega_d t)\sin(\phi)\right]\,dt\,, \end{aligned}$$

where the trigonometric identity $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ was used. Now the integral of $\sin^2(\omega_d t)$ over a cycle is $T/2$ and the integral of $\sin(\omega_d t)\cos(\omega_d t)$ over a cycle is zero, so $E_{\mathcal{E}} = \frac{1}{2}I\mathcal{E}_m\cos\phi$.

(d) The rate of energy dissipation in a resistor is given by

$$P_R = i^2 R = I^2 \sin^2(\omega_d t - \phi)$$

and the energy dissipated over a cycle is

$$E_R = I^2 \int_0^T \sin^2(\omega_d t - \phi)\,dt = \tfrac{1}{2}I^2 RT\,.$$

(e) Now $\mathcal{E}_m = IZ$, where Z is the impedance, and $R = Z\cos\phi$, so $E_{\mathcal{E}} = \frac{1}{2}I^2 TZ\cos\phi = \frac{1}{2}I^2 RT = E_R$.