Chapter 30

5

The magnitude of the magnetic field inside the solenoid is $B = \mu_0 n i_s$, where n is the number of turns per unit length and i_s is the current. The field is parallel to the solenoid axis, so the flux through a cross section of the solenoid is $\Phi_B = A_s B = \mu_0 \pi r_s^2 n i_s$, where A_s $(= \pi r_s^2)$ is the cross-sectional area of the solenoid. Since the magnetic field is zero outside the solenoid, this is also the flux through the coil. The emf in the coil has magnitude

$$\mathcal{E} = \frac{N d\Phi_B}{dt} = \mu_0 \pi r_s^2 N n \frac{di_s}{dt}$$

and the current in the coil is

$$i_c = \frac{\mathcal{E}}{R} = \frac{\mu_0 \pi r_s^2 N n}{R} \frac{di_s}{dt},$$

where N is the number of turns in the coil and R is the resistance of the coil. The current changes linearly by 3.0 A in 50 ms, so $di_s/dt = (3.0\,\text{A})/(50 \times 10^{-3}\,\text{s}) = 60\,\text{A/s}$. Thus

$$i_c = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})\pi(0.016\,\text{m})^2(120)(220 \times 10^2\,\text{m}^{-1})}{5.3\,\Omega}(60\,\text{A/s}) = 3.0 \times 10^{-2}\,\text{A}.$$

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(a) In the region of the smaller loop, the magnetic field produced by the larger loop may be taken to be uniform and equal to its value at the center of the smaller loop, on the axis. Eq. 29–26, with $z = x$ and much greater than R, gives

$$B = \frac{\mu_0 i R^2}{2x^3}$$

for the magnitude. The field is upward in the diagram. The magnetic flux through the smaller loop is the product of this field and the area (πr^2) of the smaller loop:

$$\Phi_B = \frac{\pi \mu_0 i r^2 R^2}{2x^3}.$$

(b) The emf is given by Faraday's law:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\frac{d}{dt}\left(\frac{1}{x^3}\right) = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\left(-\frac{3}{x^4}\frac{dx}{dt}\right) = \frac{3\pi\mu_0 i r^2 R^2 v}{2x^4}.$$

(c) The field of the larger loop is upward and decreases with distance away from the loop. As the smaller loop moves away, the flux through it decreases. The induced current is directed so as to produce a magnetic field that is upward through the smaller loop, in the same direction as the field of the larger loop. It is counterclockwise as viewed from above, in the same direction as the current in the larger loop.

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Thermal energy is generated at the rate $\mathcal{E}^2/R$, where $\mathcal{E}$ is the emf in the wire and R is the resistance of the wire. The resistance is given by $R = \rho L/A$, where ρ is the resistivity of copper, L is the length of the wire, and A is the cross-sectional area of the wire. The resistivity can be found in Table 26–1. Thus

$$R = \frac{\rho L}{A} = \frac{(1.69 \times 10^{-8}\,\Omega \cdot \mathrm{m})(0.500\,\mathrm{m})}{\pi(0.500 \times 10^{-3}\,\mathrm{m})^2} = 1.076 \times 10^{-2}\,\Omega\,.$$

Faraday's law is used to find the emf. If B is the magnitude of the magnetic field through the loop, then $\mathcal{E} = A\,dB/dt$, where A is the area of the loop. The radius r of the loop is $r = L/2\pi$ and its area is $\pi r^2 = \pi L^2/4\pi^2 = L^2/4\pi$. Thus

$$\mathcal{E} = \frac{L^2}{4\pi}\frac{dB}{dt} = \frac{(0.500\,\mathrm{m})^2}{4\pi}(10.0 \times 10^{-3}\,\mathrm{T/s}) = 1.989 \times 10^{-4}\,\mathrm{V}\,.$$

The rate of thermal energy generation is

$$P = \frac{\mathcal{E}^2}{R} = \frac{(1.989 \times 10^{-4}\,\mathrm{V})^2}{1.076 \times 10^{-2}\,\Omega} = 3.68 \times 10^{-6}\,\mathrm{W}\,.$$

37

(a) The field point is inside the solenoid, so Eq. 30–25 applies. The magnitude of the induced electric field is

$$E = \frac{1}{2}\frac{dB}{dt}r = \frac{1}{2}(6.5 \times 10^{-3}\,\mathrm{T/s})(0.0220\,\mathrm{m}) = 7.15 \times 10^{-5}\,\mathrm{V/m}\,.$$

(b) Now the field point is outside the solenoid and Eq. 30–27 applies. The magnitude of the induced field is

$$E = \frac{1}{2}\frac{dB}{dt}\frac{R^2}{r} = \frac{1}{2}(6.5 \times 10^{-3}\,\mathrm{T/s})\frac{(0.0600\,\mathrm{m})^2}{(0.0820\,\mathrm{m})} = 1.43 \times 10^{-4}\,\mathrm{V/m}\,.$$

51

Starting with zero current when the switch is closed, at time $t = 0$, the current in an RL series circuit at a later time t is given by

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)\,,$$

where τ_L is the inductive time constant, $\mathcal{E}$ is the emf, and R is the resistance. You want to calculate the time t for which $i = 0.9990\mathcal{E}/R$. This means

$$0.9990\frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)\,,$$

so

$$0.9990 = 1 - e^{-t/\tau_L}$$

or

$$e^{-t/\tau_L} = 0.0010 .$$

Take the natural logarithm of both sides to obtain $-(t/\tau_L) = \ln(0.0010) = -6.91$. That is, 6.91 inductive time constants must elapse.

55

(a) If the battery is switched into the circuit at time $t = 0$, then the current at a later time t is given by

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) ,$$

where $\tau_L = L/R$. You want to find the time for which $i = 0.800\mathcal{E}/R$. This means

$$0.800 = 1 - e^{-t/\tau_L}$$

or

$$e^{-t/\tau_L} = 0.200 .$$

Take the natural logarithm of both sides to obtain $-(t/\tau_L) = \ln(0.200) = -1.609$. Thus

$$t = 1.609\tau_L = \frac{1.609L}{R} = \frac{1.609(6.30 \times 10^{-6}\,\mathrm{H})}{1.20 \times 10^3\,\Omega} = 8.45 \times 10^{-9}\,\mathrm{s} .$$

(b) At $t = 1.0\tau_L$ the current in the circuit is

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-1.0}\right) = \left(\frac{14.0\,\mathrm{V}}{1.20 \times 10^3\,\Omega}\right)\left(1 - e^{-1.0}\right) = 7.37 \times 10^{-3}\,\mathrm{A} .$$

59

(a) Assume i is from left to right through the closed switch. Let i_1 be the current in the resistor and take it to be downward. Let i_2 be the current in the inductor and also take it to be downward. The junction rule gives $i = i_1 + i_2$ and the loop rule gives $i_1 R - L(di_2/dt) = 0$. Since $di/dt = 0$, the junction rule yields $(di_1/dt) = -(di_2/dt)$. Substitute into the loop equation to obtain

$$L\frac{di_1}{dt} + i_1 R = 0 .$$

This equation is similar to Eq. 30–44, and its solution is the function given as Eq. 30–45:

$$i_1 = i_0 e^{-Rt/L} ,$$

where i_0 is the current through the resistor at $t = 0$, just after the switch is closed. Now, just after the switch is closed, the inductor prevents the rapid build-up of current in its branch, so at that time, $i_2 = 0$ and $i_1 = i$. Thus $i_0 = i$, so

$$i_1 = ie^{-Rt/L}$$

and

$$i_2 = i - i_1 = i\left[1 - e^{-Rt/L}\right] .$$

(b) When $i_2 = i_1$,

$$e^{-Rt/L} = 1 - e^{-Rt/L} ,$$

so

$$e^{-Rt/L} = \frac{1}{2} .$$

Take the natural logarithm of both sides and use $\ln(1/2) = -\ln 2$ to obtain $(Rt/L) = \ln 2$ or

$$t = \frac{L}{R} \ln 2 .$$

<u>63</u>

(a) If the battery is applied at time $t = 0$, the current is given by

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) ,$$

where $\mathcal{E}$ is the emf of the battery, R is the resistance, and τ_L is the inductive time constant. In terms of R and the inductance L, $\tau_L = L/R$. Solve the current equation for the time constant. First obtain

$$e^{-t/\tau_L} = 1 - \frac{iR}{\mathcal{E}} ,$$

then take the natural logarithm of both sides to obtain

$$-\frac{t}{\tau_L} = \ln\left[1 - \frac{iR}{\mathcal{E}}\right] .$$

Since

$$\ln\left[1 - \frac{iR}{\mathcal{E}}\right] = \ln\left[1 - \frac{(2.00 \times 10^{-3}\,\mathrm{A})(10.0 \times 10^3\,\Omega)}{50.0\,\mathrm{V}}\right] = -0.5108 ,$$

the inductive time constant is $\tau_L = t/0.5108 = (5.00 \times 10^{-3}\,\mathrm{s})/(0.5108) = 9.79 \times 10^{-3}\,\mathrm{s}$ and the inductance is

$$L = \tau_L R = (9.79 \times 10^{-3}\,\mathrm{s})(10.0 \times 10^3\,\Omega) = 97.9\,\mathrm{H} .$$

(b) The energy stored in the coil is

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}(97.9\,\mathrm{H})(2.00 \times 10^{-3}\,\mathrm{A})^2 = 1.96 \times 10^{-4}\,\mathrm{J} .$$

<u>69</u>

(a) At any point, the magnetic energy density is given by $u_B = B^2/2\mu_0$, where B is the magnitude of the magnetic field at that point. Inside a solenoid, $B = \mu_0 n i$, where n is the number of turns per unit length and i is the current. For the solenoid of this problem, $n = (950)/(0.850\,\mathrm{m}) = 1.118 \times 10^3\,\mathrm{m}^{-1}$. The magnetic energy density is

$$u_B = \frac{1}{2}\mu_0 n^2 i^2 = \frac{1}{2}(4\pi \times 10^{-7}\,\mathrm{T \cdot m/A})(1.118 \times 10^3\,\mathrm{m}^{-1})^2(6.60\,\mathrm{A})^2 = 34.2\,\mathrm{J/m^3}\,.$$

(b) Since the magnetic field is uniform inside an ideal solenoid, the total energy stored in the field is $U_B = u_B V$, where V is the volume of the solenoid. V is calculated as the product of the cross-sectional area and the length. Thus

$$U_B = (34.2\,\mathrm{J/m^3})(17.0 \times 10^{-4}\,\mathrm{m^2})(0.850\,\mathrm{m}) = 4.94 \times 10^{-2}\,\mathrm{J}\,.$$

<u>**73**</u>

(a) The mutual inductance M is given by

$$\mathcal{E}_1 = M\,\frac{di_2}{dt}\,,$$

where $\mathcal{E}_1$ is the emf in coil 1 due to the changing current i_2 in coil 2. Thus

$$M = \frac{\mathcal{E}_1}{di_2/dt} = \frac{25.0 \times 10^{-3}\,\mathrm{V}}{15.0\,\mathrm{A/s}} = 1.67 \times 10^{-3}\,\mathrm{H}\,.$$

(b) The flux linkage in coil 2 is

$$N_2\Phi_{21} = Mi_1 = (1.67 \times 10^{-3}\,\mathrm{H})(3.60\,\mathrm{A}) = 6.01 \times 10^{-3}\,\mathrm{Wb}\,.$$

<u>**75**</u>

(a) Assume the current is changing at the rate di/dt and calculate the total emf across both coils. First consider the left-hand coil. The magnetic field due to the current in that coil points to the left. So does the magnetic field due to the current in coil 2. When the current increases, both fields increase and both changes in flux contribute emf's in the same direction. Thus the emf in coil 1 is

$$\mathcal{E}_1 = -\,(L_1 + M)\,\frac{di}{dt}\,.$$

The magnetic field in coil 2 due to the current in that coil points to the left, as does the field in coil 2 due to the current in coil 1. The two sources of emf are again in the same direction and the emf in coil 2 is

$$\mathcal{E}_2 = -\,(L_2 + M)\,\frac{di}{dt}\,.$$

The total emf across both coils is

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 = -\,(L_1 + L_2 + 2M)\,\frac{di}{dt}\,.$$

This is exactly the emf that would be produced if the coils were replaced by a single coil with inductance $L_{\mathrm{eq}} = L_1 + L_2 + 2M$.

(b) Reverse the leads of coil 2 so the current enters at the back of the coil rather than the front as pictured in the diagram. Then the field produced by coil 2 at the site of coil 1 is opposite the field produced by coil 1 itself. The fluxes have opposite signs. An increasing current in coil 1 tends to increase the flux in that coil but an increasing current in coil 2 tends to decrease it. The emf across coil 1 is

$$\mathcal{E}_1 = -(L_1 - M)\frac{di}{dt}.$$

Similarly the emf across coil 2 is

$$\mathcal{E}_2 = -(L_2 - M)\frac{di}{dt}.$$

The total emf across both coils is

$$\mathcal{E} = -(L_1 + L_2 - 2M)\frac{di}{dt}.$$

This the same as the emf that would be produced by a single coil with inductance $L_{eq} = L_1 + L_2 - 2M$.

79

(a) The electric field lines are circles that are concentric with the cylindrical region and the magnitude of the field is uniform around any circle. Thus the emf around a circle of radius r is $\mathcal{E} = \oint \vec{E} \cdot d\vec{s} = 2\pi r E$. Here r is inside the cylindrical region so the magnetic flux is $\pi r^2 B$. According to Faraday's law $2\pi r E = -\pi r^2 (dB/dt)$ and

$$E = -\tfrac{1}{2}r\frac{dB}{dt} = -\tfrac{1}{2}(0.050\,\text{m})(-10 \times 10^{-3}\,\text{T/s}) = 2.5 \times 10^{-4}\,\text{V/m}.$$

Since the normal used to compute the flux was taken to be into the page, in the direction of the magnetic field, the positive direction for the electric is clockwise. The calculated value of E is positive, so the electric field at point a is toward the left and $\vec{E} = -(2.5 \times 10^{-4}\,\text{V/m})\,\hat{\imath}$.

The force on the electron is $\vec{F} = -e\vec{E}$ and, according to Newton's second law, its acceleration is

$$\vec{a} = \frac{\vec{F}}{m} = -\frac{e\vec{E}}{m} = -\frac{(1.60 \times 10^{-19}\,\text{C})(-2.5 \times 10^{-4}\,\text{V/m})\,\hat{\imath}}{9.11 \times 10^{-31}\,\text{kg}} = (4.4 \times 10^{7}\,\text{m/s}^2)\,\hat{\imath}.$$

The mass and charge of an electron can be found in Appendix B.

(b) The electric field at $r = 0$ is zero, so the force and acceleration of an election placed at point b are zero.

(c) The electric field at point c has the same magnitude as the field at point a but now the field is to the right. That is $\vec{E} = (2.5 \times 10^{-4}\,\text{V/m})\,\hat{\imath}$ and $\vec{a} = -(4.4 \times 10^{7}\,\text{m/s}^2)\,\hat{\imath}$.

81

(a) The magnetic flux through the loop is $\Phi_B = BA$, where B is the magnitude of the magnetic field and A is the area of the loop. The magnitude of the average emf is given by Faraday's law : $\mathcal{E}_{avg} = B\Delta A/\Delta t$, where ΔA is the change in the area in time Δt. Since the final area is zero, the change in area is the initial area and $\mathcal{E}_{avg} = BA/\Delta t = (2.0\,\text{T})(0.20\,\text{m})^2/(0.20\,\text{s}) = 0.40\,\text{V}$.

(b) The average current in the loop is the emf divided by the resistance of the loop: $i_{\text{avg}} = \mathcal{E}_{\text{avg}}/R = (0.40\,\text{V})/(20 \times 10^{-3}\,\Omega) = 20\,\text{A}$.

85

(a), (b), (c), (d), and (e) Just after the switch is closed the current i_2 through the inductor is zero. The loop rule applied to the left loop gives $\mathcal{E} - I_1 R_1 = 0$, so $i_1 = \mathcal{E}/R_1 = (10\,\text{V})/(5.0\,\Omega) = 2.0\,\text{A}$. The junction rule gives $i_s = i_1 = 2.0\,\text{A}$. Since $i_2 = 0$, the potential difference across R_2 is $V_2 = i_2 R_2 = 0$. The potential differences across the inductor and resistor must sum to $\mathcal{E}$ and, since $V_2 = 0$, $V_L = \mathcal{E} = 10\,\text{V}$. The rate of change of i_2 is $di_2/dt = V_L/L = (10\,\text{V})/(5.0\,\text{H}) = 2.0\,\text{A/s}$.

(g), (h), (i), (j), (k), and (l) After the switch has been closed for a long time the current i_2 reaches a constant value. Since its derivative is zero the potential difference across the inductor is $V_L = 0$. The potential differences across both R_1 and R_2 are equal to the emf of the battery, so $i_1 = \mathcal{E}/R_1 = (10\,\text{V})/(5.0\,\Omega) = 2.0\,\text{A}$ and $i_2 = \mathcal{E}/R_2 = (10\,\text{V})/(10\,\Omega) = 1.0\,\text{A}$. The junction rule gives $i_s = i_1 + i_2 = 3.0\,\text{A}$.

95

(a) Because the inductor is in series with the battery the current in the circuit builds slowly and just after the switch is closed it is zero.

(b) Since all currents are zero just after the switch is closed the emf of the inductor must match the emf of the battery in magnitude. Thus $L(di_{\text{bat}}/dt) = \mathcal{E}$ and $di_{\text{bat}} = \mathcal{E}/L = (40\,\text{V})/(50 \times 10^{-3}\,\text{H}) = 8.0 \times 10^2\,\text{A/s}$.

(c) Replace the two resistors in parallel with their equivalent resistor. The equivalent resistance is

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(20\,\text{k}\Omega)(20\,\text{k}\Omega)}{20\,\text{k}\Omega + 20\,\text{k}\Omega} = 10\,\text{k}\Omega\,.$$

The current as a function of time is given by

$$i_{\text{bat}} = \frac{\mathcal{E}}{R_{\text{eq}}}\left[1 - e^{-t/\tau_L}\right],$$

where τ_L is the inductive time constant. Its value is $\tau_L = L/R_{\text{eq}} = (50 \times 10^{-3}\,\text{H})/(10 \times 10^3\,\Omega) = 5.0 \times 10^{-6}\,\text{s}$. At $t = 3.0 \times 10^{-6}\,\text{s}$, $t/\tau_L = (3.0)/(5.0) = 0.60$ and

$$i_{\text{bat}} = \frac{40\,\text{V}}{10 \times 10^3\,\Omega}\left[1 - e^{-0.60}\right] = 1.8 \times 10^{-3}\,\text{A}\,.$$

(d) Differentiate the expression for i_{bat} to obtain

$$\frac{di_{\text{bat}}}{dt} = \frac{\mathcal{E}}{R_{\text{eq}}}\frac{1}{\tau_L}e^{-t/\tau_L} = \frac{\mathcal{E}}{L}e^{-t/\tau_L},$$

where $\tau_L = L/R_{\text{eq}}$ was used to obtain the last form. At $t = 3.0 \times 10^{-6}\,\text{s}$

$$\frac{di_{\text{bat}}}{dt} = \frac{40\,\text{V}}{50 \times 10^{-3}\,\text{H}}e^{-0.60} = 4.4 \times 10^2\,\text{A/s}\,.$$

(e) A long time after the switch is closed the currents are constant and the emf of the inductor is zero. The current in the battery is $i_{\text{bat}} = \mathcal{E}/R_{\text{eq}} = (40\,\text{V})/(10 \times 10^3\,\Omega) = 4.0 \times 10^{-3}\,\text{A}$.

(f) The currents are constant and $di_{\text{bat}}/dt = 0$.

97

(a) and (b) Take clockwise current to be positive and counterclockwise current to be negative. Then according to the right-hand rule we must take the normal to the loop to be into the page, so the flux is negative if the magnetic field is out of the page and positive if it is into the page. Assume the field in region 1 is out of the page. We will obtain a negative result for the field if the assumption is incorrect. Let x be the distance that the front edge of the loop is into region 1. Then while the loop is entering this region flux is $-B_1Hx$ and, according to Faraday's law, the emf induced around the loop is $\mathcal{E} = B_1H(dx/dt) = B_1Hv$. The current in the loop is $i = \mathcal{E}/R = B_1Hv/R$, so

$$B_1 = \frac{iR}{Hv} = \frac{(3.0 \times 10^{-6}\,\text{A})(0.020\,\Omega)}{(0.0150\,\text{m})(0.40\,\text{m/s})} = 1.0 \times 10^{-5}\,\text{T}\,.$$

The field is positive and therefore out of the page.

(c) and (d) Assume that the field B_2 of region 2 is out of the page. Let x now be the distance the front end of the loop is into region 2 as the loop enters that region. The flux is $-B_1H(D-x) - B_2Hx$, the emf is $\mathcal{E} = -B_1Hv + B_2Hv = (B_2 - B_1)Hv$, and the current is $i = (B_2 - B_1)Hv/R$. The field of region 2 is

$$B_2 = B_1 + \frac{iR}{Hv} = 1.0 \times 10^{-5}\,\text{T} + \frac{(-2.0 \times 10^{-6}\,\text{A}(0.020\,\Omega)}{(0.015\,\text{m})(0.40\,\text{m/s})} = 3.3 \times 10^{-6}\,\text{T}\,.$$

The field is positive, indicating that it is out of the page.