Chapter 27

7

(a) Let i be the current in the circuit and take it to be positive if it is to the left in R_1. Use Kirchhoff's loop rule: $\mathcal{E}_1 - iR_2 - iR_1 - \mathcal{E}_2 = 0$. Solve for i:

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{12\,\text{V} - 6.0\,\text{V}}{4.0\,\Omega + 8.0\,\Omega} = 0.50\,\text{A}\,.$$

A positive value was obtained, so the current is counterclockwise around the circuit.

(b) and (c) If i is the current in a resistor with resistance R, then the power dissipated by that resistor is given by $P = i^2R$. For R_1 the power dissipated is

$$P_1 = (0.50\,\text{A})^2(4.0\,\Omega) = 1.0\,\text{W}$$

and for R_2 the power dissipated is

$$P_2 = (0.50\,\text{A})^2(8.0\,\Omega) = 2.0\,\text{W}\,.$$

(d) and (e) If i is the current in a battery with emf $\mathcal{E}$, then the battery supplies energy at the rate $P = i\mathcal{E}$ provided the current and emf are in the same direction. The battery absorbs energy at the rate $P = i\mathcal{E}$ if the current and emf are in opposite directions. For battery 1 the power is

$$P_1 = (0.50\,\text{A})(12\,\text{V}) = 6.0\,\text{W}$$

and for battery 2 it is

$$P_2 = (0.50\,\text{A})(6.0\,\text{V}) = 3.0\,\text{W}\,.$$

(f) and (g) In battery 1, the current is in the same direction as the emf so this battery supplies energy to the circuit. The battery is discharging. The current in battery 2 is opposite the direction of the emf, so this battery absorbs energy from the circuit. It is charging.

13

(a) If i is the current and ΔV is the potential difference, then the power absorbed is given by $P = i\,\Delta V$. Thus

$$\Delta V = \frac{P}{i} = \frac{50\,\text{W}}{1.0\,\text{A}} = 50\,\text{V}\,.$$

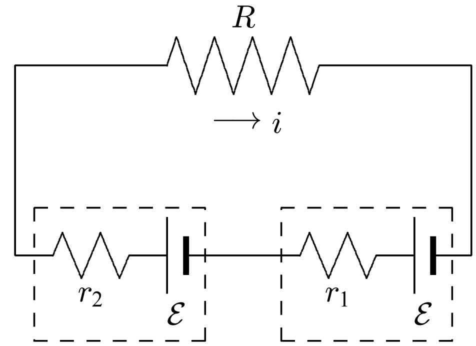
Since energy is absorbed, point A is at a higher potential than point B; that is, $V_A - V_B = 50\,\text{V}$.

(b) The end-to-end potential difference is given by $V_A - V_B = +iR + \mathcal{E}$, where $\mathcal{E}$ is the emf of element C and is taken to be positive if it is to the left in the diagram. Thus $\mathcal{E} = V_A - V_B - iR = 50\,\text{V} - (1.0\,\text{A})(2.0\,\Omega) = 48\,\text{V}$.

(c) A positive value was obtained for $\mathcal{E}$, so it is toward the left. The negative terminal is at B.

21

(a) and (b) The circuit is shown in the diagram to the right. The current is taken to be positive if it is clockwise. The potential difference across battery 1 is given by $V_1 = \mathcal{E} - ir_1$ and for this to be zero, the current must be $i = \mathcal{E}/r_1$. Kirchhoff's loop rule gives $2\mathcal{E} - ir_1 - ir_2 - iR = 0$. Substitute $i = \mathcal{E}/r_1$ and solve for R. You should get $R = r_1 - r_2 = 0.016\,\Omega - 0.012\,\Omega = 0.004\,\Omega$.



Now assume that the potential difference across battery 2 is zero and carry out the same analysis. You should find $R = r_2 - r_1$. Since $r_1 > r_2$ and R must be positive, this situation is not possible. Only the potential difference across the battery with the larger internal resistance can be made to vanish with the proper choice of R.

29

Let r be the resistance of each of the thin wires. Since they are in parallel, the resistance R of the composite can be determined from

$$\frac{1}{R} = \frac{9}{r},$$

or $R = r/9$. Now

$$r = \frac{4\rho\ell}{\pi d^2}$$

and

$$R = \frac{4\rho\ell}{\pi D^2},$$

where ρ is the resistivity of copper. Here $\pi d^2/4$ was used for the cross-sectional area of any one of the original wires and $\pi D^2/4$ was used for the cross-sectional area of the replacement wire. Here d and D are diameters. Since the replacement wire is to have the same resistance as the composite,

$$\frac{4\rho\ell}{\pi D^2} = \frac{4\rho\ell}{9\pi d^2}.$$

Solve for D and obtain $D = 3d$.

33

Replace the two resistors on the left with their equivalent resistor. They are in parallel, so the equivalent resistance is $R_{eq} = 1.0\,\Omega$. The circuit now consists of the two emf devices and four resistors. Take the current to be upward in the right-hand emf device. Then the loop rule gives $\mathcal{E}_2 - iR_{eq} - 3iR - \mathcal{E}_2$, where $R = 2.0\,\Omega$. The current is

$$i = \frac{\mathcal{E}_2 - \mathcal{E}_1}{R_{eq} + 3R} = \frac{12\,\text{V} - 5.0\,\text{V}}{1.0\,\Omega + 3(2.0\,\Omega)} = 1.0\,\text{A}.$$

To find the potential at point 1 take a path from ground, through the equivalent resistor and $\mathcal{E}_2$, to the point. The result is $V_1 = iR_{\text{eq}} - \mathcal{E}_1 = (1.0\,\text{A})(1.0\,\Omega) - 12\,\text{V} = -11\,\text{V}$. To find the potential at point 2 continue the path through the lowest resistor on the digram. It is $V_2 = V_1 + iR = -11\,\text{V} + (1.0\,\text{A})(2.0\,\Omega) = -9.0\,\text{V}$.

47

(a) and (b) The copper wire and the aluminum jacket are connected in parallel, so the potential difference is the same for them. Since the potential difference is the product of the current and the resistance, $i_C R_C = i_A R_A$, where i_C is the current in the copper, i_A is the current in the aluminum, R_C is the resistance of the copper, and R_A is the resistance of the aluminum. The resistance of either component is given by $R = \rho L/A$, where ρ is the resistivity, L is the length, and A is the cross-sectional area. The resistance of the copper wire is

$$R_C = \frac{\rho_C L}{\pi a^2}$$

and the resistance of the aluminum jacket is

$$R_A = \frac{\rho_A L}{\pi(b^2 - a^2)} .$$

Substitute these expressions into $i_C R_C = i_A R_A$ and cancel the common factors L and π to obtain

$$\frac{i_C \rho_C}{a^2} = \frac{i_A \rho_A}{b^2 - a^2} .$$

Solve this equation simultaneously with $i = i_C + i_A$, where i is the total current. You should get

$$i_C = \frac{a^2 \rho_C i}{(b^2 - a^2)\rho_C + a^2 \rho_A}$$

and

$$i_A = \frac{(b^2 - a^2)\rho_C i}{(b^2 - a^2)\rho_C + a^2 \rho_A} .$$

The denominators are the same and each has the value

$$\begin{aligned}(b^2 - a^2)\rho_C + a^2 \rho_A &= \left[(0.380 \times 10^{-3}\,\text{m})^2 - (0.250 \times 10^{-3}\,\text{m})^2\right](1.69 \times 10^{-8}\,\Omega\cdot\text{m}) \\ &\quad + (0.250 \times 10^{-3}\,\text{m})^2(2.75 \times 10^{-8}\,\Omega\cdot\text{m}) \\ &= 3.10 \times 10^{-15}\,\Omega\cdot\text{m}^3 .\end{aligned}$$

Thus

$$i_C = \frac{(0.250 \times 10^{-3}\,\text{m})^2(2.75 \times 10^{-8}\Omega\cdot\text{m})(2.00\,\text{A})}{3.10 \times 10^{-15}\,\Omega\cdot\text{m}^3} = 1.11\,\text{A}$$

and

$$\begin{aligned}i_A &= \frac{\left[(0.380 \times 10^{-3}\,\text{m})^2 - (0.250 \times 10^{-3}\,\text{m})^2\right](1.69 \times 10^{-8}\,\Omega\cdot\text{m})(2.00\,\text{A})}{3.10 \times 10^{-15}\,\Omega\cdot\text{m}^3} \\ &= 0.893\,\text{A} .\end{aligned}$$

(c) Consider the copper wire. If V is the potential difference, then the current is given by $V = i_C R_C = i_C \rho_C L/\pi a^2$, so

$$L = \frac{\pi a^2 V}{i_C \rho_C} = \frac{\pi(0.250 \times 10^{-3}\,\text{m})^2(12.0\,\text{V})}{(1.11\,\text{A})(1.69 \times 10^{-8}\,\Omega\cdot\text{m})} = 126\,\text{m}\,.$$

57

During charging the charge on the positive plate of the capacitor is given by Eq. 27–33, with $RC = \tau$. That is,

$$q = C\mathcal{E}\left[1 - e^{-t/\tau}\right]\,,$$

where C is the capacitance, $\mathcal{E}$ is applied emf, and τ is the time constant. You want the time for which $q = 0.990C\mathcal{E}$, so

$$0.990 = 1 - e^{-t/\tau}\,.$$

Thus

$$e^{-t/\tau} = 0.010\,.$$

Take the natural logarithm of both sides to obtain $t/\tau = -\ln 0.010 = 4.61$ and $t = 4.61\tau$.

65

(a), (b), and (c) At $t = 0$, the capacitor is completely uncharged and the current in the capacitor branch is as it would be if the capacitor were replaced by a wire. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is downward. Let i_3 be the current in R_3 and take it to be positive if it is downward. The junction rule produces $i_1 = i_2 + i_3$, the loop rule applied to the left-hand loop produces

$$\mathcal{E} - i_1 R_1 - i_2 R_2 = 0\,,$$

and the loop rule applied to the right-hand loop produces

$$i_2 R_2 - i_3 R_3 = 0\,.$$

Since the resistances are all the same, you can simplify the mathematics by replacing R_1, R_2, and R_3 with R. The solution to the three simultaneous equations is

$$i_1 = \frac{2\mathcal{E}}{3R} = \frac{2(1.2 \times 10^3\,\text{V})}{3(0.73 \times 10^6\,\Omega)} = 1.1 \times 10^{-3}\,\text{A}$$

and

$$i_2 = i_3 = \frac{\mathcal{E}}{3R} = \frac{1.2 \times 10^3\,\text{V}}{3(0.73 \times 10^6\,\Omega)} = 5.5 \times 10^{-4}\,\text{A}\,.$$

(d), (e), and (f) At $t = \infty$, the capacitor is fully charged and the current in the capacitor branch is zero. Then $i_1 = i_2$ and the loop rule yields

$$\mathcal{E} - i_1 R_1 - i_1 R_2 = 0\,.$$

The solution is

$$i_1 = i_2 = \frac{\mathcal{E}}{2R} = \frac{1.2 \times 10^3\,\text{V}}{2(0.73 \times 10^6\,\Omega)} = 8.2 \times 10^{-4}\,\text{A}\,.$$

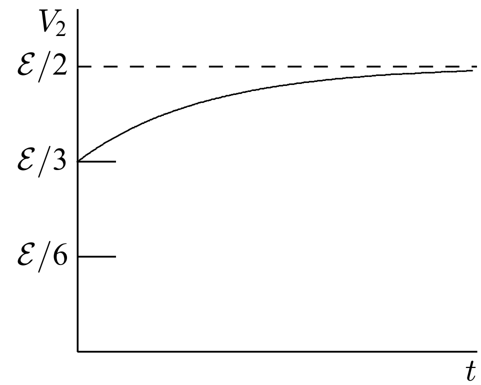
(g) and (h) The potential difference across resistor 2 is $V_2 = i_2 R_2$. At $t = 0$ it is

$$V_2 = (5.5 \times 10^{-4}\,\text{A})(0.73 \times 10^6\,\Omega) = 4.0 \times 10^2\,\text{V}$$

and at $t = \infty$ it is

$$V_2 = (8.2 \times 10^{-4}\,\text{A})(0.73 \times 10^6\,\Omega) = 6.0 \times 10^2\,\text{V}\,.$$

(i) The graph of V_2 versus t is shown to the right.



73

As the capacitor discharges the potential difference across its plates at time t is given by $V = V_0 e^{-t/\tau}$, where V_0 is the potential difference at time $t = 0$ and τ is the capacitive time constant. This equation is solved for the time constant, with result

$$\tau = -\frac{t}{\ln(V/V_0)}\,.$$

Since the time constant is $\tau = RC$, where RR is the resistance and C is the capacitance,

$$R = -\frac{t}{C\ln(V/V_0)}\,.$$

For the smaller time interval

$$R = -\frac{10.0 \times 10^{-6}\,\text{s}}{(0.220 \times 10^{-6}\,\text{F})\ln\left(\dfrac{0.800\,\text{V}}{5.00\,\text{V}}\right)} = 24.8\,\Omega\,.$$

and for the larger time interval

$$R = -\frac{6.00 \times 10^{-3}\,\text{s}}{(0.220 \times 10^{-6}\,\text{F})\ln\left(\dfrac{0.800\,\text{V}}{5.00\,\text{V}}\right)} = 1.49 \times 10^4\,\Omega\,.$$

75

(a) Let i be the current, which is the same in both wires, and $\mathcal{E}$ be the applied potential difference. Then the loop equation gives $\mathcal{E} - iR_A - iR_B = 0$ and the current is

$$i = \frac{\mathcal{E}}{R_A + R_B} = \frac{60.0\,\text{V}}{0.127\,\Omega + 0.729\,\Omega} = 70.1\,\text{A}\,.$$

The current density in wire A is

$$J_A = \frac{i}{\pi r_A^2} = \frac{70.1\,\text{A}}{\pi(1.30 \times 10^{-3}\,\text{m})^2} = 1.32 \times 10^7\,\text{A/m}^2\,.$$

(b) The potential difference across wire A is $V_A = iR_A = (70.1\,\text{A})(0.127\,\Omega) = 8.90\,\text{V}$.

(c) The resistance is $R_A = \rho_A L/A$, where ρ is the resistivity, A is the cross-sectional area, and L is the length. The resistivity of wire A is

$$\rho_A = \frac{R_A A}{L} = \frac{(0.127\,\Omega)\pi(1.30 \times 10^{-3}\,\text{m})^2}{40.0\,\text{m}} = 1.69 \times 10^{-8}\,\Omega \cdot \text{m}\,.$$

According to Table 26–1 the material is copper.

(d) Since wire B has the same diameter and length as wire A and carries the same current, the current density in it is the same, $1.32 \times 10^7\,\text{A/m}^2$.

(e) The potential difference across wire B is $V_B = iR_B = (70.1\,\text{A})(0.729\,\Omega) = 51.1\,\text{V}$.

(f) The resistivity of wire B is

$$\rho_B = \frac{R_B A}{L} = \frac{(0.729\,\Omega)\pi(1.30 \times 10^{-3}\,\text{m})^2}{40.0\,\text{m}} = 9.68 \times 10^{-8}\,\Omega \cdot \text{m}\,.$$

According to Table 26–1 the material is iron.

77

The three circuit elements are in series, so the current is the same in all of them. Since the battery is discharging, the potential difference across its terminals is $V_{\text{batt}} = \mathcal{E} - ir$, where $\mathcal{E}$ is its emf and r is its internal resistance. Thus

$$r = \frac{\mathcal{E} - V}{i} = \frac{12\,\text{V} - 11.4\,\text{V}}{50\,\text{A}} = 0.012\,\Omega\,.$$

This is less than $0.0200\,\Omega$, so the battery is not defective.

The resistance of the cable is $R_{\text{cable}} = V_{\text{cable}}/i = (3.0\,\text{V})/(50\,\text{A} = 0.060\,\Omega$, which is greater than $0.040\,\Omega$. The cable is defective.

The potential difference across the motor is $V_{\text{motor}} = 11.4\,\text{V} - 3.0\,\text{V} = 8.4\,\text{V}$ and its resistance is $R_{\text{motor}} = V_{\text{motor}}/i = (8.4\,\text{V})/(50\,\text{A}) = 0.17\,\Omega$, which is less than $0.200\,\Omega$. The motor is not defective.

85

Let R_{S0} be the resistance of the silicon resistor at 20° and R_{I0} be the resistance of the iron resistor at that temperature. At some other temperature T the resistance of the silicon resistor is $R_S = R_{S0} + \alpha_S R_{S0}(T - 20°\text{C})$ and the resistance of the iron resistor is $R_I = R_{I0} + \alpha_I R_{I0}(T - 20°\text{C})$. Here α_S and α_I are the temperature coefficients of resistivity. The resistors are series so the resistance of the combination is

$$R = R_{S0} + R_{I0} + (\alpha_S R_{S0} + \alpha_I R_{I0})(T - 20°\text{C})\,.$$

We want $R_{S0} + R_{I0}$ to be $1000\,\Omega$ and $\alpha_S R_{S0} + \alpha_I R_{I0}$ to be zero. Then the resistance of the combination will be independent of the temperature.

The second equation gives $R_{I0} = -(\alpha_S/\alpha_I)R_{S0}$ and when this is used to substitute for R_{I0} in the first equation the result is $R_{S0} - (\alpha_S/\alpha_I)R_{S0} = 1000\,\Omega$. The solution for R_{S0} is

$$R_{S0} = \frac{1000\,\Omega}{\dfrac{\alpha_S}{\alpha_I} - 1} = \frac{1000\,\Omega}{\dfrac{-70 \times 10^{-3}\,\mathrm{K}^{-1}}{6.5 \times 10^{-3}\,\mathrm{K}^{-1}} - 1} = 85\,\Omega\,,$$

where values for the temperature coefficients of resistivity were obtained from Table 26–1. The resistance of the iron resistor is $R_{I0} = 1000\,\Omega - 85\,\Omega = 915\,\Omega$.

95

When the capacitor is fully charged the potential difference across its plates is $\mathcal{E}$ and the energy stored in it is $U = \frac{1}{2}C\mathcal{E}^2$.

(a) The current is given as a function of time by $i = (\mathcal{E}/R)e^{-t/\tau}$, where τ (= RC) is the capacitive time constant. The rate with which the emf device supplies energy is $P_\mathcal{E} = i\mathcal{E}$ and the energy supplied in fully charging the capacitor is

$$E_\mathcal{E} = \int_0^\infty P_\mathcal{E}\,dt = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-t/\tau}\,dt = \frac{\mathcal{E}^2\tau}{R} = \frac{\mathcal{E}^2 RC}{R} = C\mathcal{E}^2\,.$$

This is twice the energy stored in the capacitor.

(b) The rate with which energy is dissipated in the resistor is $P_R = i^2R$ and the energy dissipated as the capacitor is fully charged is

$$E_R = \int_0^\infty P_R\,dt = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-2t/\tau}\,dt = \frac{\mathcal{E}^2\tau}{2R} = \frac{\mathcal{E}^2 RC}{2R} = \tfrac{1}{2}C\mathcal{E}^2\,.$$

97

(a) Immediately after the switch is closed the capacitor is uncharged and since the charge on the capacitor is given by $q = CV_C$, the potential difference across its plates is zero. Apply the loop rule to the right-hand loop to find that the potential difference across R_2 must also be zero. Now apply the loop rule to the left-hand loop to find that $\mathcal{E} - i_1R_1 = 0$ and $i_1 = \mathcal{E}/R_1 = (30\,\mathrm{V})/(20 \times 10^3\,\Omega) = 1.5 \times 10^{-3}\,\mathrm{A}$.

(b) Since the potential difference across R_2 is zero and this potential difference is given by $V_{R2} = i_2R_2$, $i_2 = 0$.

(c) A long time later, when the capacitor is fully charged, the current is zero in the capacitor branch and the current is the same in the two resistors. The loop rule applied to the left-hand loop gives $\mathcal{E} - iR_1 - iR_2 = 0$, so $i = \mathcal{E}/(R_1 + R_2) = (30\,\mathrm{V})/(20 \times 10^3\,\Omega + 10 \times 10^3\,\Omega) = 1.0 \times 10^{-3}\,\mathrm{A}$.

99

(a) R_2 and R_3 are in parallel, with an equivalent resistance of $R_2R_3/(R_2 + R_3)$, and this combination is in series with R_1, so the circuit can be reduced to a single loop with an emf $\mathcal{E}$ and a resistance $R_\mathrm{eq} = R_1 + R_2R_3/(R_2 + R_3) = (R_1R_2 + R_1R_3 + R_2R_3)/(R_1 + R_2)$. The current is

$$i = \frac{\mathcal{E}}{R_\mathrm{eq}} = \frac{(R_2 + R_3)\mathcal{E}}{R_1R_2 + R_1R_3 + R_2R_3}\,.$$

The rate with which the battery supplies energy is

$$P = i\mathcal{E} = \frac{(R_2 + R_3)\mathcal{E}^2}{R_1R_2 + R_1R_3 + R_2R_3}.$$

The derivative with respect to R_3 is

$$\frac{dP}{dR_3} = \frac{\mathcal{E}^2}{R_1R_2 + R_1R_3 + R_2R_3} - \frac{(R_2 + R_3)(R_1 + R_2)\mathcal{E}^2}{(R_1R_2 + R_1R_3 + R_2R_3)^2} = -\frac{\mathcal{E}^2 R_2^2}{(R_1R_2 + R_1R_3 + R_2R_3)^2},$$

where the last form was obtained with a little algebra. The derivative is negative for all (positive) values of the resistances, so P has its maximum value for $R_3 = 0$.

(b) Substitute $R_3 = 0$ in the expression for P to obtain

$$P = \frac{R_1\mathcal{E}^2}{R_1R_2} = \frac{\mathcal{E}^2}{R_1} = \frac{12.0\,\text{V}}{10.0\,\Omega} = 14.4\,\text{W}.$$

<u>101</u>

If the batteries are connected in series the total emf in the circuit is $N\mathcal{E}$ and the equivalent resistance is $R + nr$, so the current is $i = N\mathcal{E}/(R + Nr)$. If $R = r$, then $i = N\mathcal{E}/(N+1)r$.

If the batteries are connected in parallel then the emf in the circuit is $\mathcal{E}$ and the equivalent resistance is $R + r/N$, so the current is $i = \mathcal{E}/(R + r/N) = N\mathcal{E}/(NR + r)$. If $R = r$, $i = N\mathcal{E}/(N+1)r$, the same as when they are connected in series.