Chapter 26

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(a) The magnitude of the current density is given by $J = nqv_d$, where n is the number of particles per unit volume, q is the charge on each particle, and v_d is the drift speed of the particles. The particle concentration is $n = 2.0 \times 10^8\,\text{cm}^{-3} = 2.0 \times 10^{14}\,\text{m}^{-3}$, the charge is $q = 2e = 2(1.60 \times 10^{-19}\,\text{C}) = 3.20 \times 10^{-19}\,\text{C}$, and the drift speed is $1.0 \times 10^5\,\text{m/s}$. Thus

$$J = (2 \times 10^{14}\,\text{m}^{-3})(3.2 \times 10^{-19}\,\text{C})(1.0 \times 10^5\,\text{m/s}) = \boxed{6.4\,\text{A/m}^2}\,.$$

(b) Since the particles are positively charged, the current density is in the same direction as their motion, to the north.

(c) The current cannot be calculated unless the cross-sectional area of the beam is known. Then $i = JA$ can be used.

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The resistance of the wire is given by $R = \rho L/A$, where ρ is the resistivity of the material, L is the length of the wire, and A is the cross-sectional area of the wire. The cross-sectional area is $A = \pi r^2 = \pi(0.50 \times 10^{-3}\,\text{m})^2 = 7.85 \times 10^{-7}\,\text{m}^2$. Here $r = 0.50\,\text{mm} = 0.50 \times 10^{-3}\,\text{m}$ is the radius of the wire. Thus

$$\rho = \frac{RA}{L} = \frac{(50 \times 10^{-3}\,\Omega)(7.85 \times 10^{-7}\,\text{m}^2)}{2.0\,\text{m}} = 2.0 \times 10^{-8}\,\Omega \cdot \text{m}\,.$$

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The resistance of the coil is given by $R = \rho L/A$, where L is the length of the wire, ρ is the resistivity of copper, and A is the cross-sectional area of the wire. Since each turn of wire has length $2\pi r$, where r is the radius of the coil, $L = (250)2\pi r = (250)(2\pi)(0.12\,\text{m}) = 188.5\,\text{m}$. If r_w is the radius of the wire, its cross-sectional area is $A = \pi r_w^2 = \pi(0.65\times10^{-3}\,\text{m})^2 = 1.33\times10^{-6}\,\text{m}^2$. According to Table 26–1, the resistivity of copper is $1.69 \times 10^{-8}\,\Omega \cdot \text{m}$. Thus

$$R = \frac{\rho L}{A} = \frac{(1.69 \times 10^{-8}\,\Omega \cdot \text{m})(188.5\,\text{m})}{1.33 \times 10^{-6}\,\text{m}^2} = 2.4\,\Omega\,.$$

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Since the mass and density of the material do not change, the volume remains the same. If L_0 is the original length, L is the new length, A_0 is the original cross-sectional area, and A is the new cross-sectional area, then $L_0A_0 = LA$ and $A = L_0A_0/L = L_0A_0/3L_0 = A_0/3$. The new resistance is

$$R = \frac{\rho L}{A} = \frac{\rho 3L_0}{A_0/3} = 9\frac{\rho L_0}{A_0} = 9R_0\,,$$

where R_0 is the original resistance. Thus $R = 9(6.0\,\Omega) = 54\,\Omega$.

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The resistance of conductor A is given by

$$R_A = \frac{\rho L}{\pi r_A^2},$$

where r_A is the radius of the conductor. If r_o is the outside radius of conductor B and r_i is its inside radius, then its cross-sectional area is $\pi(r_o^2 - r_i^2)$ and its resistance is

$$R_B = \frac{\rho L}{\pi(r_o^2 - r_i^2)}.$$

The ratio is

$$\frac{R_A}{R_B} = \frac{r_o^2 - r_i^2}{r_A^2} = \frac{(1.0\,\text{mm})^2 - (0.50\,\text{mm})^2}{(0.50\,\text{mm})^2} = 3.0\,.$$

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(a) Electrical energy is transferred to internal energy at a rate given by

$$P = \frac{V^2}{R},$$

where V is the potential difference across the heater and R is the resistance of the heater. Thus

$$P = \frac{(120\,\text{V})^2}{14\,\Omega} = 1.0 \times 10^3\,\text{W} = 1.0\,\text{kW}\,.$$

(b) The cost is given by

$$C = (1.0\,\text{kW})(5.0\,\text{h})(\$0.050\,/\text{kW}\cdot\text{h}) = \$0.25\,.$$

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(a) Let P be the rate of energy dissipation, i be the current in the heater, and V be the potential difference across the heater. They are related by $P = iV$. Solve for i:

$$i = \frac{P}{V} = \frac{1250\,\text{W}}{115\,\text{V}} = 10.9\,\text{A}\,.$$

(b) According to the definition of resistance $V = iR$, where R is the resistance of the heater. Solve for R:

$$R = \frac{V}{i} = \frac{115\,\text{V}}{10.9\,\text{A}} = 10.6\,\Omega\,.$$

(c) The thermal energy E produced by the heater in time t (= 1.0 h = 3600 s) is

$$E = Pt = (1250\,\text{W})(3600\,\text{s}) = 4.5 \times 10^6\,\text{J}\,.$$

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(a) and (b) Calculate the electrical resistances of the wires. Let ρ_C be the resistivity of wire C, r_C be its radius, and L_C be its length. Then the resistance of this wire is

$$R_C = \rho_C \frac{L_C}{\pi r_C^2} = (2.0 \times 10^{-6}\,\Omega\cdot\mathrm{m})\frac{1.0\,\mathrm{m}}{\pi(0.50\times 10^{-3}\,\mathrm{m})^2} = 2.54\,\Omega\,.$$

Let ρ_D be the resistivity of wire D, r_D be its radius, and L_D be its length. Then the resistance of this wire is

$$R_D = \rho_D \frac{L_D}{\pi r_D^2} = (1.0 \times 10^{-6}\,\Omega\cdot\mathrm{m})\frac{1.0\,\mathrm{m}}{\pi(0.25\times 10^{-3}\,\mathrm{m})^2} = 5.09\,\Omega\,.$$

If i is the current in the wire, the potential difference between points 1 and 2 is

$$\Delta V_{12} = iR_C = (2.0\,\mathrm{A})(2.54\,\Omega) = 5.1\,\mathrm{V}$$

and the potential difference between points 2 and 3 is

$$\Delta V_{23} = iR_D = (2.0\,\mathrm{A})(5.09\,\Omega) = 10\,\mathrm{V}\,.$$

(c) and (d) The rate of energy dissipation between points 1 and 2 is

$$P_{12} = i^2 R_C = (2.0\,\mathrm{A})^2(2.54\,\Omega) = 10\,\mathrm{W}$$

and the rate of energy dissipation between points 2 and 3 is

$$P_{23} = i^2 R_D = (2.0\,\mathrm{A})^2(5.09\,\Omega) = 20\,\mathrm{W}\,.$$

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(a) The charge that strikes the surface in time Δt is given by $\Delta q = i\,\Delta t$, where i is the current. Since each particle carries charge $2e$, the number of particles that strike the surface is

$$N = \frac{\Delta q}{2e} = \frac{i\,\Delta t}{2e} = \frac{(0.25\times 10^{-6}\,\mathrm{A})(3.0\,\mathrm{s})}{2(1.6\times 10^{-19}\,\mathrm{C})} = 2.3\times 10^{12}\,.$$

(b) Now let N be the number of particles in a length L of the beam. They will all pass through the beam cross section at one end in time $t = L/v$, where v is the particle speed. The current is the charge that moves through the cross section per unit time. That is, $i = 2eN/t = 2eNv/L$. Thus, $N = iL/2ev$.

Now find the particle speed. The kinetic energy of a particle is

$$K = 20\,\mathrm{MeV} = (20\times 10^6\,\mathrm{eV})(1.60\times 10^{-19}\,\mathrm{J/eV}) = 3.2\times 10^{-12}\,\mathrm{J}\,.$$

Since $K = \frac{1}{2}mv^2$, $v = \sqrt{2K/m}$. The mass of an alpha particle is four times the mass of a proton or $m = 4(1.67\times 10^{-27}\,\mathrm{kg}) = 6.68\times 10^{-27}\,\mathrm{kg}$, so

$$v = \sqrt{\frac{2(3.2\times 10^{-12}\,\mathrm{J})}{6.68\times 10^{-27}\,\mathrm{kg}}} = 3.1\times 10^7\,\mathrm{m/s}$$

and

$$N = \frac{iL}{2ev} = \frac{(0.25 \times 10^{-6}\,\mathrm{A})(20 \times 10^{-2}\,\mathrm{m})}{2(1.60 \times 10^{-19}\,\mathrm{C})(3.1 \times 10^{7}\,\mathrm{m/s})} = 5.0 \times 10^{3}\,.$$

(c) Use conservation of energy. The initial kinetic energy is zero, the final kinetic energy is $20\,\mathrm{MeV} = 3.2 \times 10^{-12}\,\mathrm{J}$, the initial potential energy is $qV = 2eV$, and the final potential energy is zero. Here V is the electric potential through which the particles are accelerated. Conservation of energy leads to $K_f = U_i = 2eV$, so

$$V = \frac{K_f}{2e} = \frac{3.2 \times 10^{-12}\,\mathrm{J}}{2(1.60 \times 10^{-19}\,\mathrm{C})} = 10 \times 10^{6}\,\mathrm{V}\,.$$

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Let R_H be the resistance at the higher temperature (800° C) and let R_L be the resistance at the lower temperature (200° C). Since the potential difference is the same for the two temperatures, the rate of energy dissipation at the lower temperature is $P_L = V^2/R_L$, and the rate of energy dissipation at the higher temperature is $P_H = V^2/R_H$, so $P_L = (R_H/R_L)P_H$. Now $R_L = R_H + \alpha R_H\,\Delta T$, where ΔT is the temperature difference $T_L - T_H = -600°\,\mathrm{C}$. Thus,

$$P_L = \frac{R_H}{R_H + \alpha R_H\,\Delta T}\,P_H = \frac{P_H}{1 + \alpha\,\Delta T} = \frac{500\,\mathrm{W}}{1 + (4.0 \times 10^{-4}\,/°\mathrm{C})(-600°\,\mathrm{C})} = 660\,\mathrm{W}\,.$$

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If the resistivity is ρ_0 at temperature T_0, then the resistivity at temperature T is $\rho = \rho_0 + \alpha\rho_0(T - T_0)$, where α is the temperature coefficient of resistivity. The solution for T is

$$T = \frac{\rho - \rho_0 + \alpha\rho_0 T_0}{\alpha\rho_0}\,.$$

Substitute $\rho = 2\rho_0$ to obtain

$$T = T_0 + \frac{1}{\alpha} = 20.0°\mathrm{C} + \frac{1}{4.3 \times 10^{-3}\,\mathrm{K}^{-1}} = 250°\mathrm{C}\,.$$

The value of α was obtained from Table 26–1.