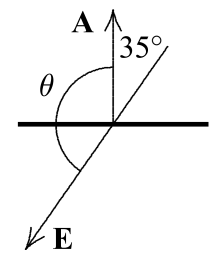
Chapter 23

1

The vector area $\vec{A}$ and the electric field $\vec{E}$ are shown on the diagram to the right. The angle θ between them is $180° - 35° = 145°$, so the electric flux through the area is $\Phi = \vec{E} \cdot \vec{A} = EA\cos\theta = (1800\,\mathrm{N/C})(3.2 \times 10^{-3}\,\mathrm{m})^2 \cos 145° = -1.5 \times 10^{-2}\,\mathrm{N \cdot m^2/C}$.



9

Let A be the area of one face of the cube, E_u be the magnitude of the electric field at the upper face, and E_ℓ be the magnitude of the field at the lower face. Since the field is downward, the flux through the upper face is negative and the flux through the lower face is positive. The flux through the other faces is zero, so the total flux through the cube surface is $\Phi = A(E_\ell - E_u)$. The net charge inside the cube is given by Gauss' law:

$$\begin{aligned} q = \epsilon_0\Phi = \epsilon_0 A(E_\ell - E_u) &= (8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2})(100\,\mathrm{m})^2(100\,\mathrm{N/C} - 60.0\,\mathrm{N/C}) \\ &= 3.54 \times 10^{-6}\,\mathrm{C} = 3.54\,\mu\mathrm{C}\,. \end{aligned}$$

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(a) The charge on the surface of the sphere is the product of the surface charge density σ and the surface area of the sphere ($4\pi r^2$, where r is the radius). Thus

$$q = 4\pi r^2\sigma = 4\pi\left(\frac{1.2\,\mathrm{m}}{2}\right)^2(8.1 \times 10^{-6}\,\mathrm{C/m^2}) = 3.7 \times 10^{-5}\,\mathrm{C}\,.$$

(b) Choose a Gaussian surface in the form a sphere, concentric with the conducting sphere and with a slightly larger radius. The flux through the surface is given by Gauss' law:

$$\Phi = \frac{q}{\epsilon_0} = \frac{3.7 \times 10^{-5}\,\mathrm{C}}{8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}} = 4.1 \times 10^6\,\mathrm{N \cdot m^2/C}\,.$$

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The magnitude of the electric field produced by a uniformly charged infinite line is $E = \lambda/2\pi\epsilon_0 r$, where λ is the linear charge density and r is the distance from the line to the point where the field is measured. See Eq. 23–12. Thus

$$\lambda = 2\pi\epsilon_0 Er = 2\pi(8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2})(4.5 \times 10^4\,\mathrm{N/C})(2.0\,\mathrm{m}) = 5.0 \times 10^{-6}\,\mathrm{C/m}\,.$$

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Assume the charge density of both the conducting rod and the shell are uniform. Neglect fringing. Symmetry can be used to show that the electric field is radial, both between the rod and the shell and outside the shell. It is zero, of course, inside the rod and inside the shell since they are conductors.

(a) and (b) Take the Gaussian surface to be a cylinder of length L and radius r, concentric with the conducting rod and shell and with its curved surface outside the shell. The area of the curved surface is $2\pi rL$. The field is normal to the curved portion of the surface and has uniform magnitude over it, so the flux through this portion of the surface is $\Phi = 2\pi rLE$, where E is the magnitude of the field at the Gaussian surface. The flux through the ends is zero. The charge enclosed by the Gaussian surface is $Q_1 - 2.00Q_1 = -Q_1$. Gauss' law yields $2\pi r\epsilon_0 LE = -Q_1$, so

$$E = -\frac{Q_1}{2\pi\epsilon_0 Lr} = -\frac{(8.99 \times 10^9\,N \cdot \mathrm{m^2/C^2})(3.40 \times 10^{-12}\,\mathrm{C})}{(11.00\,\mathrm{m})(26.0 \times 10^{-3}\,\mathrm{m})} = -0.214\,\mathrm{N/C}\,.$$

The magnitude of the field is $0.214\,\mathrm{N/C}$. The negative sign indicates that the field points inward.

(c) and (d) Take the Gaussian surface to be a cylinder of length L and radius r, concentric with the conducting rod and shell and with its curved surface between the conducting rod and the shell. As in (a), the flux through the curved portion of the surface is $\Phi = 2\pi rLE$, where E is the magnitude of the field at the Gaussian surface, and the flux through the ends is zero. The charge enclosed by the Gaussian surface is only the charge Q_1 on the conducting rod. Gauss' law yields $2\pi\epsilon_0 rLE = Q_1$, so

$$E = \frac{Q_1}{2\pi\epsilon_0 Lr} = \frac{2(8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2})(3.40 \times 10^{-12}\,\mathrm{C})}{(11.00\,\mathrm{m})(6.50 \times 10^{-3}\,\mathrm{m})} = +0.855\,\mathrm{N/C}\,.$$

The positive sign indicates that the field points outward.

(e) Consider a Gaussian surface in the form of a cylinder of length L with the curved portion of its surface completely within the shell. The electric field is zero at all points on the curved surface and is parallel to the ends, so the total electric flux through the Gaussian surface is zero and the net charge within it is zero. Since the conducting rod, which is inside the Gaussian cylinder, has charge Q_1, the inner surface of the shell must have charge $-Q_1 = -3.40 \times 10^{-12}\,\mathrm{C}$.

(f) Since the shell has total charge $-2.00Q_1$ and has charge $-Q_1$ on its inner surface, it must have charge $-Q_1 = -3.40 \times 10^{-12}\,\mathrm{C}$ on its outer surface.

35

(a) To calculate the electric field at a point very close to the center of a large, uniformly charged conducting plate, we may replace the finite plate with an infinite plate with the same area charge density and take the magnitude of the field to be $E = \sigma/\epsilon_0$, where σ is the area charge density for the surface just under the point. The charge is distributed uniformly over both sides of the original plate, with half being on the side near the field point. Thus

$$\sigma = \frac{q}{2A} = \frac{6.0 \times 10^{-6}\,\mathrm{C}}{2(0.080\,\mathrm{m})^2} = 4.69 \times 10^{-4}\,\mathrm{C/m^2}\,.$$

The magnitude of the field is

$$E = \frac{\sigma}{\epsilon_0} = \frac{4.69 \times 10^{-4}\,\mathrm{C/m^2}}{8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}} = 5.3 \times 10^7\,\mathrm{N/C}\,.$$

The field is normal to the plate and since the charge on the plate is positive, it points away from the plate.

(b) At a point far away from the plate, the electric field is nearly that of a point particle with charge equal to the total charge on the plate. The magnitude of the field is $E = q/4\pi\epsilon_0 r^2$, where r is the distance from the plate. Thus

$$E = \frac{(8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2})(6.0 \times 10^{-6}\,\mathrm{C})}{(30\,\mathrm{m})^2} = 60\,\mathrm{N/C}\,.$$

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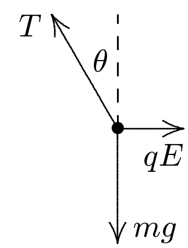

The forces on the ball are shown in the diagram to the right. The gravitational force has magnitude mg, where m is the mass of the ball; the electrical force has magnitude qE, where q is the charge on the ball and E is the electric field at the position of the ball; and the tension in the thread is denoted by T. The electric field produced by the plate is normal to the plate and points to the right. Since the ball is positively charged, the electric force on it also points to the right. The tension in the thread makes the angle θ (= 30°) with the vertical.

Since the ball is in equilibrium the net force on it vanishes. The sum of the horizontal components yields $qE - T\sin\theta = 0$ and the sum of the vertical components yields $T\cos\theta - mg = 0$. The expression $T = qE/\sin\theta$, from the first equation, is substituted into the second to obtain $qE = mg\tan\theta$.

The electric field produced by a large uniform plane of charge is given by $E = \sigma/2\epsilon_0$, where σ is the surface charge density. Thus

$$\frac{q\sigma}{2\epsilon_0} = mg\tan\theta$$

and

$$\begin{aligned}\sigma &= \frac{2\epsilon_0 mg\tan\theta}{q} \\ &= \frac{2(8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2})(1.0 \times 10^{-6}\,\mathrm{kg})(9.8\,\mathrm{m/s^2})\tan 30^\circ}{2.0 \times 10^{-8}\,\mathrm{C}} \\ &= 5.0 \times 10^{-9}\,\mathrm{C/m^2}\,.\end{aligned}$$

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Charge is distributed uniformly over the surface of the sphere and the electric field it produces at points outside the sphere is like the field of a point particle with charge equal to the net charge on the sphere. That is, the magnitude of the field is given by $E = q/4\pi\epsilon_0 r^2$, where q is the magnitude of the charge on the sphere and r is the distance from the center of the sphere to the point where the field is measured. Thus

$$q = 4\pi\epsilon_0 r^2 E = \frac{(0.15\,\mathrm{m})^2(3.0\times10^3\,\mathrm{N/C})}{8.99\times10^9\,\mathrm{N\cdot m^2/C^2}} = 7.5\times10^{-9}\,\mathrm{C}\,.$$

The field points inward, toward the sphere center, so the charge is negative: $-7.5\times10^{-9}\,\mathrm{C}$.

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To find an expression for the electric field inside the shell in terms of A and the distance from the center of the shell, select A so the field does not depend on the distance.

Use a Gaussian surface in the form of a sphere with radius r_g, concentric with the spherical shell and within it ($a < r_g < b$). Gauss' law is used to find the magnitude of the electric field a distance r_g from the shell center.

The charge that is both in the shell and within the Gaussian sphere is given by the integral $q_{\mathrm{enc}} = \int \rho\,\mathrm{d}V$ over the portion of the shell within the Gaussian surface. Since the charge distribution has spherical symmetry, we may take $\mathrm{d}V$ to be the volume of a spherical shell with radius r and infinitesimal thickness $\mathrm{d}r$: $\mathrm{d}V = 4\pi r^2\,\mathrm{d}r$. Thus

$$q_{\mathrm{enc}} = 4\pi\int_a^{r_g}\rho r^2\,\mathrm{d}r = 4\pi\int_a^{r_g}\frac{A}{r}r^2\,\mathrm{d}r = 4\pi A\int_a^{r_g} r\,\mathrm{d}r = 2\pi A(r_g^2 - a^2)\,.$$

The total charge inside the Gaussian surface is $q + q_{\mathrm{enc}} = q + 2\pi A(r_g^2 - a^2)$.

The electric field is radial, so the flux through the Gaussian surface is $\Phi = 4\pi r_g^2 E$, where E is the magnitude of the field. Gauss' law yields

$$4\pi\epsilon_0 E r_g^2 = q + 2\pi A(r_g^2 - a^2)\,.$$

Solve for E:

$$E = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r_g^2} + 2\pi A - \frac{2\pi A a^2}{r_g^2}\right]\,.$$

For the field to be uniform, the first and last terms in the brackets must cancel. They do if $q - 2\pi A a^2 = 0$ or $A = q/2\pi a^2 = (45.0\times10^{-15}\,\mathrm{C})/2\pi(2.00\times10^{-2}\,\mathrm{m})^2 = 1.79\times10^{-11}\,\mathrm{C/m^2}$.

59

(a) The magnitude E_1 of the electric field produced by the charge q on the spherical shell is $E_1 = q/4\pi\epsilon_0 R_o^2$, where R_o is the radius of the outer surface of the shell. Thus

$$q = 4\pi\epsilon_0 E_1 R_o^2 = \frac{(450\,\mathrm{N/C})(0.20\,\mathrm{m})^2}{8.99\times10^9\,\mathrm{N\cdot m^2/C^2}} = 2.0\times10^{-9}\,\mathrm{C}\,.$$

(b) Since the field at P is outward and is reduced in magnitude the field of Q must be inward. Q is a negative charge and the magnitude of its field at P is $E_2 = 450\,\mathrm{N/C} - 180\,\mathrm{N/C} = 270\,\mathrm{N/C}$. The value of Q is

$$Q = 4\pi\epsilon_0 E_2 R_o^2 = -\frac{(270\,\mathrm{N/C})(0.20\,\mathrm{m})^2}{8.99\times10^9\,\mathrm{N\cdot m^2/C)^2}} = -1.2\times10^{-9}\,\mathrm{C}\,.$$

(c) Gauss' law tells us that since the electric field is zero inside a conductor the net charge inside a spherical surface with a radius that is slightly larger than the inside radius of the shell must be zero. Thus the charge on the inside surface of the shell is $+1.2 \times 10^{-9}$ C.

(d) The remaining charge on the shell must be on its outer surface and this is $2.0 \times 10^{-9}\,\text{C} - 1.2 \times 10^{-9}\,\text{C} = +0.8 \times 10^{-9}\,\text{C}$.

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(a) Draw a spherical Gaussian surface with radius r, concentric with the shells. The electric field, if it exists, is radial and so is normal to the surface. The integral in Gauss' law is $\oint \vec{E} \cdot d\vec{A} = 4\pi r^2 E$, where E is the radial component of the field. For $r < a$ then charge enclosed is zero. Gauss' law gives $4\pi r^2 E = 0$, so $E = 0$.

(b) For $a < r < b$ the charge enclosed by the Gaussian surface is q_a, so the law gives $4\pi r^2 E = q_a/\epsilon_0$ and $E = q_a/4\pi\epsilon_0 r^2$.

(c) For $r > b$ the charge enclosed by the Gaussian surface is $q_a + q_b$, so $4\pi\epsilon_0 E = (q_a + q_b)/\epsilon_0$ and $E = (q_a + q_b)/4\pi\epsilon_0 r^2$.

(d) Consider first a spherical Gaussian with radius just slightly greater than a. The electric field is zero everywhere on this surface, so according to Gauss' law it encloses zero net charge. Since there is no charge in the cavity the charge on the inner surface of the smaller shell is zero. The total charge on the smaller shell is q_a and this must reside on the outer surface. Now consider a spherical Gaussian surface with radius slight larger than the inner radius of the larger shell. This surface also encloses zero net charge, which is the sum of the charge on the outer surface of the smaller shell and the charge on the inner surface of the larger shell. Thus the charge on the inner surface of the larger shell is $-q_a$. The net charge on the larger shell is q_b, with $-q_a$ on its inner surface, so the charge on its outer surface must be $q_b - (-q_a) = q_b + q_a$.

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(a) The magnitude of the electric field due to a large uniformly charged plate is given by $\sigma/2\epsilon_0$, where σ is the surface charge density. In the region between the oppositely charged plates the fields of the plates are in the same direction, so the net field has magnitude $E = \sigma/\epsilon_0$. The electrical force on an electron has magnitude $eE = e\sigma/\epsilon_0$ and the gravitational force on it is mg, where m is it mass. If these forces are to balance, they must have the same magnitude, so $mg = e\sigma/\epsilon_0$ and

$$\sigma = \frac{mg\epsilon_0}{e} = \frac{(9.11 \times 10^{-31}\,\text{kg})(9.8\,\text{m/s}^2)(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}{1.60 \times 10^{-19}\,\text{C}} = 4.9 \times 10^{-22}\,\text{C/m}^2\,.$$

(b) The gravitational force is downward, so the electrical force must be upward. Since an electron is negatively charged the electrical force on it is opposite to the electric field, so the electric field must be downward.

79

(a) Let Q be the net charge on the shell, q_i be the charge on its inner surface and q_o be the charge on its outer surface. Then $Q = q_i + q_o$ and $q_i = Q - q_o = (-10\,\mu\text{C}) - (-14\,\mu\text{C}) = +4\,\mu\text{C}$.

(b) Let q be the charge on the particle. Gauss' law tells us that since the electric field is zero inside the conducting shell the net charge inside any spherical surface that entirely within the shell is zero. Thus the sum of the charge on the particle and on the inner surface of the shell is zero, so $q + q_i = 0$ and $q = -q_i = -4\,\mu$C.